Section 2.2
Determinant of a matrix
Using row operations

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MATHS 211: Linear Algebra

Goal:

1. To define the determinant of a matrix.
2. To find the determinant of a matrix using cofactor expansion (Section 2.1).
3. To find the determinant of a matrix using row reduction (Section 2.2).
4. Explore the properties of the determinant and its relation to the inverse. (Section 2.3)
5. To solve linear system using the Cramer's rule. (Section 2.3)

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Theorem 1

If A is an $n \times n$ triangular matrix (upper triangular, lower triangular, or diagonal), then $\det(A)$ is the product of the entries on the main diagonal of the matrix, that is $\det(A) = a_{11}a_{22}\dots a_{nn}$.

Theorem 2

(Row operations and determinant) If A is an $n \times n$ matrix.

1. *If $B \sim A$ by multiplying a row of A by k, then*

$$\det(B) = k\det(A)$$

2. *If $B \sim A$ by exchanging two rows of A, then*

$$\det(B) = -\det(A)$$

3. *If $B \sim A$ by adding a multiple of one row to another row of A, then*

Example 3

Find $\det(A)$ for

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -3 & 3 & 5 \\ 7 & 0 & 6 \end{pmatrix}$$

Solution:

Example 4

Find $\det(A)$ for

$$A = \begin{pmatrix} 5 & 2 & -2 & 0 \\ 3 & 2 & -2 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & -1 & 5 & 7 \end{pmatrix}$$

Solution: