Section 4.4
Basis

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MATHS 211: Linear Algebra

Goal:

1. Define a basis of a vector space.
2. Coordinates relative to a Basis.

Basis

Definition 1

Let V be a vector space. $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is called a **basis** are if

1. $\mathcal{B}$ is linearly independent.
2. $\mathcal{B}$ spans V.

Example 2

Determine whether the vectors $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ form a basis for $\mathbb{R}^3$ or not.

This is called the *standard* basis for $\mathbb{R}^3$.

Example 3

Show that the vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$ form a basis for $\mathbb{R}^3$.

Homework 4

Example 4

Determine whether the vectors $\mathbf{v}_1 = 2 - 4X + X^2$, $\mathbf{v}_2 = 3 + 2X - X^2$, $\mathbf{v}_3 = 1 + 6X - 2X^2$ form a basis for P_2.

Example 5

Determine whether the vectors $\mathbf{v}_1 = \begin{pmatrix} 3 & 4 \\ 3 & -4 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 0 & -8 \\ -12 & -2 \end{pmatrix}$ form a basis for $\mathrm{Mat}(2, 2, \mathbb{R})$.

Theorem 6

Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ *be a basis for* V*, then every vector* v *in* V *can be written* **uniquely** *in the form*

$$v = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$$

Definition 7

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be basis for V and let w in V with

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$$

The scalars $c_1, \ldots, c_n$ are called **coordinates** of $\mathbf{v}$ in terms of $\mathcal{B}$. The vector $(c_1, \ldots, c_n) \in \mathbb{R}^n$ is called the **coordinate vector of v relative to** $\mathcal{B}$. It is denoted by

$$(\mathbf{v})_{\mathcal{B}} = (c_1, c_2, \ldots, c_n)$$

Example 8

Let $\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \right\}$ be a basis for some subspace of $\mathbb{R}^3$.

Find the coordinate vector of

1. $\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix}$.
2. $\begin{pmatrix} 5 \\ -12 \\ 3 \end{pmatrix}$

Example 9

Let $\mathcal{B} = \{1 + X, 1 + X^2, X + X^2\}$ be a basis for some subspace of $\mathbb{P}_2$. Find the coordinate vector of

1. $3 - X - 2X^2$

Homework 4

Example 10

Let $\mathcal{B} = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$ be a basis for some subspace of $\text{Mat}(2, 2, \mathbb{R})$. Find the coordinate vector of

1. $\begin{pmatrix} -5 & 4 \\ 1 & -1 \end{pmatrix}$