Section 4.8
Rank and Nullity

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MATHS 211: Linear Algebra

Goal:

1. Define the rank, nullity of a matrix and ways to find them.
2. The Fundamental spaces of a matrix.

1 - Define rank and Nullity

Example 1

Consider the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & 1 \\ 3 & 5 & 7 & -6 \\ 1 & 4 & 2 & 7 \end{pmatrix}$$

1. Find the dimension of the row/column space of A.
2. Find the dimension of the null space.

Definition 2

Let A be an $m \times n$ matrix. The dimension of row/column space of A is called the **rank** of A, denoted by $\text{Rank}(A)$.

Definition 3

Let A be an $m \times n$ matrix. The dimension of null space of A is called the **nullity** of A, denoted by $\text{Nullity}(A)$.

Theorem 4

Rank(A)= Number of pivots in the RREF of A.
Nullity(A)= Number of free variables in the RREF of A.

Theorem 5

(The Dimension Theorem for Matrices)

$$Rank(A) + Nullity(A) = n = number\ of\ columns$$

Example 6

Discuss the rank and nullity of the matrix

$$A = \begin{pmatrix} 1 & -1 & t \\ 1 & -t & -1 \\ t^2 & 1 & -1 \end{pmatrix}$$

Example 7

Find the largest possible value for the rank of A and the smallest possible value for the nullity of A, given the size of A is
(a) 4×6 (b) 5×5 (c) 6×4

2 - The four fundamental spaces of a matrix

We have 6 spaces associated with a matrix A and these are

row space of A	column space of A	null space of A
row space of A^T	column space of A^T	null space of A^T

But in fact we have only **four** fundamental spaces associated with A and these are

row space of A	column space of A	null space of A
		null space of A^T

Dimensions of these spaces are

r	r	$n-r$
		$m-r$

Example 8

If the size of A and rank of A are given, find the dimension of the row space of A, column space of A, null space of A, and null space of A^T.

(a) 3×4, Rank(A)=2.
(b) 3×3, Rank(A)=1.
(c) 6×5, Rank(A)=5.
(d) 5×6, Rank(A)=2.