Section 5.2
Diagonalization

Dr. Abdulla Eid

Dr. Abdulla Eid

College of Science

MATHS 211: Linear Algebra

Goal:

1. Finding diagonalization of a matrix.
2. When has a matrix A, a diagonalization?
3. Benefits of diagonalization of a matrix.

Dr. Abdulla Eid

Example 1

Find the Diagonalization of

$$A = \begin{pmatrix} 2 & -1 \\ 10 & -9 \end{pmatrix}$$

Questions: How can we do that?

When that can happen?

Why would you that in the first place?

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Example 2

Write the following matrix

$$A = \begin{pmatrix} 3 & 0 \\ 5 & 3 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?
When that can happen?
Why would you that in the first place?

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Example 3

Write the following matrix

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?

When that can happen?

Why would you that in the first place?

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Example 4

Write the following matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

as $A = PDP^{-1}$, for some matrix P and diagonal matrix D.

Questions: How can we do that?

When that can happen?

Why would you that in the first place?

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When can we diagonalize a matrix?

Theorem 5

A is diagonalizable if and only if A has exactly n linearly independent Eigenvectors.

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A shortcut (sometimes is useful)

Theorem 6

If A has n distinct Eigenvalues, then A is diagonalizable.

Why diagonalization?

Example 7

Find A^{11}, where

$$A = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & -3 & 1 \end{pmatrix}$$

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Why diagonalization?

Example 8

Find A^{1000}, A^{-1000}, A^{2017}, A^{20}, where

$$A = \begin{pmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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Why diagonalization?

If A is diagonalizable, i.e., $A = PDP^{-1}$, then we have

1. $A^{-1} = PD^{-1}P^{-1}$.
2. $A^n = PD^nP^{-1}$.
3. $\det(A) = \det(D) =$ multiplication of the Eigenvalues.
4. $Rank(A) = Rank(PDP^{-1})$.
5. $Nullity(A) = Nullity(PDP^{-1})$.
6. $Trace(A) = Trace(PDP^{-1})$.

Dr. Abdulla Eid