University of Bahrain
Department of Mathematics
MATHS122: Calculus II
Spring 2016
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Test 1

Student's Name: ________________________ ID: ________________________

Section: ________________________ Serial Number: ________________________

- *Do not* open the exam until you are instructed to do so.
- Show sufficient work to justify each answer.
- Calculators are allowed but cell phones are *not* allowed during the exam.
- Exchange of any material such as calculator, pen, eraser is *not* allowed.
- **No** questions are allowed.
- You have 1 hour to finish this exam. You can leave only after 30 minutes of the exam.
- There are **??** questions and **??** pages in this exam.

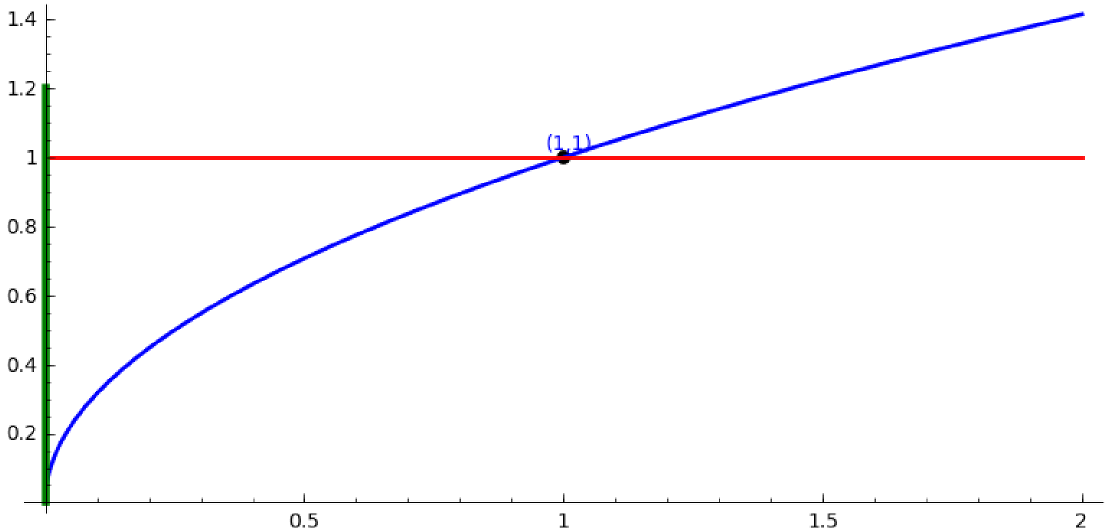
Run LaTeX again to produce the table

Question 1 (6 points)

Consider the region enclosed by the curves $y = \sqrt{x}$, $y = 1$, and $x = 0$.

(a) Sketch the region above.

Solution:



(b) Find the volume of the solid generated by revolving the region about the line $x = 4$.

Solution: If we place a vertical rectangle in the region of a distance x from the y–axis, we have

$$\text{radius} = \text{distance between the rectangle and the axis of revolution} = 4 - x$$
$$\text{height} = \text{Top - Bottom} = 1 - \sqrt{x}$$
$$\text{thickness} = dx$$

Thus by the cylindrical shell method we have

$$V = 2\pi \int (\text{radius})(\text{height})(\text{thickness})$$
$$V = 2\pi \int_0^1 (4 - x)(1 - \sqrt{x})\, dx$$
$$= 2\pi \int_0^1 4 - 4x^{\frac{1}{2}} - x + x^{\frac{3}{2}}\, dx$$
$$= 2\pi \left[4x - \frac{8}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2 + \frac{2}{5}x^{\frac{5}{2}} \right]_0^1$$
$$= \frac{37}{15}\pi$$

Question 2 (4 points)

Find the length of the curve

$$y = 24x^{\frac{3}{2}}, \qquad 0 \leqslant x \leqslant 2$$

Solution:

$$\begin{aligned} L &= \int_0^2 \sqrt{1+(y')^2}\,dx \\ &= \int_0^2 \sqrt{1+\left(36x^{\frac{1}{2}}\right)^2}\,dx \\ &= \int_0^2 \sqrt{1+1296x}\,dx \\ &= \left[\frac{1}{1296}\cdot\frac{2}{3}\,(1+1296x)^{\frac{3}{2}}\right]_0^2 \\ &= \frac{1}{1944}(2593)^{\frac{3}{2}} - \frac{1}{1944} \\ &\sim 67.92 \end{aligned}$$

Question 3 (10 points)

(i) Use l'hopital's rule to find the following limits:

(a) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$

Solution:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin x - x}{x^3} &\overset{(H)}{=} \lim_{x\to 0} \frac{\cos x - 1}{3x^2} \\ &\overset{(H)}{=} \lim_{x\to 0} \frac{-\sin x}{6x} \\ &\overset{(H)}{=} \lim_{x\to 0} \frac{-\cos x}{6} \\ &= \frac{-1}{6}\end{aligned}$$

(b) $\lim_{x\to 0^+} x\,(\ln x)^2$

Solution:

$$\begin{aligned}\lim_{x\to 0^+} x(\ln x)^2 &= \lim_{x\to 0^+} \frac{(\ln x)^2}{\frac{1}{x}} \\ &\overset{(H)}{=} \lim_{x\to 0^+} \frac{2(\ln x)\frac{1}{x}}{\frac{-1}{x^2}} \\ &= \lim_{x\to 0^+} -2x(\ln x) \\ &= \lim_{x\to 0^+} \frac{-2(\ln x)}{\frac{1}{x}} \\ &\overset{(H)}{=} \lim_{x\to 0^+} \frac{-2\frac{1}{x}}{\frac{-1}{x^2}} \\ &= \lim_{x\to 0^+} -2x \\ &= 0\end{aligned}$$

(ii) Evaluate the following integrals:

(a) $\displaystyle\int (6x+5)\cos x\,dx$

Solution: We use integration by parts. Let

$$u = 6x+5 \qquad dv = \cos x\,dx$$
$$du = 6\,dx \qquad v = \sin x$$

$$\begin{aligned}\int (6x+5)\cos x\,dx &= uv - \int v\,du \\ &= (6x+5)\sin x - \int 6\sin x\,dx \\ &= (6x+5)\sin x + 6\cos x + C\end{aligned}$$

(b) $\displaystyle\int_1^e 25x^4 \ln x\,dx$

Solution: We use integration by parts. Let

$$u = \ln x \qquad dv = 25x^4\,dx$$
$$du = \frac{1}{x}\,dx \qquad v = 5x^5$$

$$\begin{aligned}\int_1^e 25x^4\ln x\,dx &= uv - \int v\,du \\ &= \left[5x^5\ln x\right]_1^e - \int_1^e 5x^5\frac{1}{x}\,dx \\ &= \left[5x^5\ln x\right]_1^e - \int_1^e 5x^4\,dx \\ &= \left[5x^5\ln x\right]_1^e - \left[x^5\right]_1^e \\ &= 5e^5 - e^5 + 1 \\ &= 4e^5 + 1\end{aligned}$$