University of Bahrain
Department of Mathematics
MATHS122: Calculus II
Spring 2016
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Worksheet 10: Convergence Tests

Students' Name: ____

1. Determine whether the following series converge or diverges. Write the name of the test that was used.

   1. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} + \dots$

   2. $\sum_{n=2}^{\infty} \frac{\sqrt{n}+4}{n^2}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

4. $\displaystyle\sum_{n=0}^{\infty} \frac{n \sin^2 n}{1+n^3}$

5. $\displaystyle\sum_{n=3}^{\infty} \frac{tan^{-1} n}{n^2}$

6. $\displaystyle\sum_{n=2}^{\infty} \left(1+\frac{1}{n}\right)^{n} e^{-n}$

7. $\displaystyle\sum_{n=1}^{\infty} \sin\left(1+\frac{1}{n}\right)$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^{n}}$

9. $\sum_{n=2}^{\infty} \left(\frac{2n}{n+1} \right)^{5n}$

10. $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

11. What is the domain of the Riemann–zeta function ζ which is defined by

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$

2. Find the value of p such that the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$$

converges.
(Hint: Use the integral test).

3. State and prove the integral test for series.

4. Prove the following properties of series given that $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$ converge, $\sum_{n=1}^{\infty} d_n$ be any series, and c is a real number:

   1. $\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$
   2. $\sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$.
   3. $\sum_{n=1}^{\infty} d_n$ and $\sum_{n=k}^{\infty} d_n$ have the same behavior (both converge or diverge).
   4. Give an example of two divergent series $\sum_{n=1}^{\infty} a_n, \sum_{n=1}^{\infty} b_n$ such that their sum $\sum_{n=1}^{\infty} (a_n + b_n)$ converges.

5. Consider the decimal number $\alpha = 0.d_1d_2d_3\ldots$.

    1. What should be the conditions on $d_1, d_2, d_3, \ldots$?

    2. Write α as an infinite sum.

    3. Prove that your series in (2) indeed converges for all numbers.

6. If $a_n \geqslant 0$ for all n and $\sum a_n$ converges. Prove that $\sum a_n^2$ converges.