

University of Bahrain
Department of Mathematics
MATHS122: Calculus II
Spring 2016
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Worksheet 11: Power Series

Students' Name: _____

1. Find the radii and the interval of convergence of the following power series.

$$(1) \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^n}{2^n}.$$

$$(2) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{1+3^n}.$$

$$(3) \sum_{n=0}^{\infty} \frac{x^{2n}}{e^{\sqrt{n}}}.$$

$$(4) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{e^{n^2}}.$$

$$(5) \sum_{n=0}^{\infty} \frac{(-1)^n (2n)! x^{2n}}{(n!)^2}.$$

2. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n^2}$. Find the interval of convergence of $f(x)$, $f'(x)$, and $f''(x)$. (Are these should be all the same? Explain!)

3. (Give the radius of convergence for each part)

(1) Give a power series representation for $f(x) = \frac{1}{1+x}$.

(2) Use differentiation to find a power series representation for $\frac{1}{(1+x)^2}$.

(3) Use part (2) to find a power series representation for $\frac{1}{(1+x)^3}$.

(4) Use part (3) to find a power series representation for $\frac{x^2}{(1+x)^3}$.

(5) Use part (1) to find a power series representation for $\ln(1+x)$.

- (6) Use part (5) to find a power series representation for $\ln t$.
- (7) Find the power series representation for $(1 + x) \ln(1 + x) - x$.
4. Find a power series representation for $\frac{t}{1 - t^8}$ and also the definite integrals $\int \frac{t}{1 - t^8} dt$. What is the interval of convergence?

5. A function f is defined by

$$f(x) = 1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + \dots$$

Find the interval of convergence of the series and find an explicit formula for $f(x)$.