University of Bahrain
Department of Mathematics
MATHS122: Calculus II
Spring 2016
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Worksheet 5: Integration of rational functions

Students' Name: ____________________

1. Write the partial fraction decomposition of the following rational functions (**Do not evaluate the constants**).

   1. $\dfrac{2x+1}{x^2-7x+12}$

   2. $\dfrac{x^2}{(x-1)(x^2+2x+1)}$

   3. $\dfrac{48}{x^4+9x^2}$

   4. $\dfrac{x^4+x^2-1}{x^3+x}$

2. Find the integrals in Question 1. (**Do not evaluate the constants**).

1. $\displaystyle\int \frac{2x+1}{x^2-7x+12}\,dx$

2. $\displaystyle\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx$

3. $\displaystyle\int \frac{48}{x^4+9x^2}\,dx$

4. $\displaystyle\int \frac{x^4+x^2-1}{x^3+x}\,dx$

3. Find the constant in the partial fraction decomposition of the rational functions in Question 1.

   1. $\dfrac{2x+1}{x^2-7x+12}$

   2. $\dfrac{x^2}{(x-1)(x^2+2x+1)}$

3. $\dfrac{48}{x^4 + 9x^2}$

4. $\dfrac{x^4 + x^2 - 1}{x^3 + x}$

4. Evaluate the following integral by completing the square.

   1. $\int \frac{1}{x^2 - 2x}\, dx$

   2. $\int \frac{2x + 1}{4x^2 + 12x - 7}\, dx$

5. Make a substitution to express the integrand as rational function and then evaluate the integral.

   1. $\int \frac{\sqrt{x+1}}{x}\, dx$

   2. $\int \frac{1}{1+\sqrt[3]{x}}\, dx$

6. To find the integral of a rational function of $\sin x$ and $\cos x$, the German mathematician K. Weierstrass noticed that the substitution $t = \tan\left(\frac{x}{2}\right)$ will convert the rational function into ordinary rational function of t.

   1. If $t = \tan\left(\frac{x}{2}\right)$ $(-\pi < t < \pi)$, show that

   $$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}} \text{ and } \cos\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

   2. Show that

   $$\cos x = \frac{1-t^2}{1+t^2} \text{ and } \sin x = \frac{2t}{1+t^2}$$

   3. Show that

   $$dx = \frac{2}{1+t^2}\,dt$$

7. Use the idea of the previous exercise to convert the following integral into integral of ordinary rational function in t.

   1. $\displaystyle\int \frac{1}{1-\cos x}\,dx$

   2. $\displaystyle\int \frac{1}{1+\sin x-\cos x}\,dx$