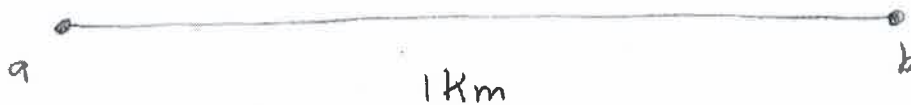


§ 10.0 - Motivation and Introduction

Consider the case that we have a path of length 1 km



and you would like either

- walk this distance
- tile the tiles
- paint the path

using the following strategy:

In each day, we walk (tile) only half of what is remaining

for example,

Day 1:



work done

$$\frac{1}{2}$$

Day 2:



$$\frac{1}{4} = \frac{1}{2^2}$$

Day 3:



$$\frac{1}{8} = \frac{1}{2^3}$$

Day 4:



$$\frac{1}{16} = \frac{1}{2^4}$$

Day n

$$\frac{1}{2^n}$$

Question : Is this work going to an End?

Mathematically speaking, No. Because whenever we have a remaining part, we will fill only half of it and thus we will have a remaining part.

Paradox : This means that I can't walk/tile/paint this distance!!!

Answer :

Yes, we can walk/tile/paint. Because at some point of time, the remaining part is so narrow that we can't break to half anymore in the nature (or we can't fill ~~half~~ half by our foot/tile/paint). So we will fill the remaining part ~~all~~ in once.

Lesson : Some mathematical ideas cannot be done in the real life as we wish them.

Total work :

Day 1 + Day 2 + Day 3 + ... + Day n + ...

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots + \frac{1}{2^n} + \dots = 1$$

infinite series, since it sums to one, we call it

Converges to 1.

Example 1:

Suppose we walk/tile/paint one km everyday

Day 1



work done

1

Day 2



1

⋮

Day n



1

So the total work done is

Day 1 Day 2 Day 3

$$1 + 1 + 1 + \dots = \infty$$

← since we will tile the whole Earth and more.

This type of series is called diverges.

Aim: Given any series, we want just to tell whether it is converges or diverges.

For that, we will develop many tests that could help us to determine that.

