§ 12.2 - Polar Coordinates

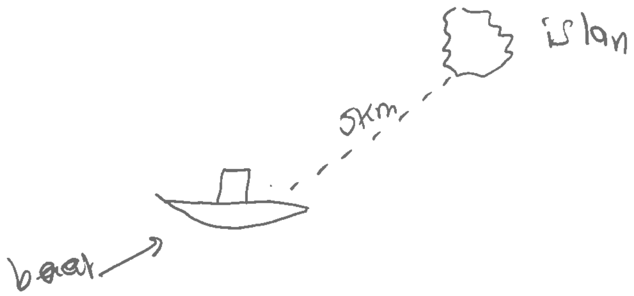
Motivation 1: How to describe a point in the plane?

Solution: one way is to give (x, y)

x: how many steps left / right from the origin

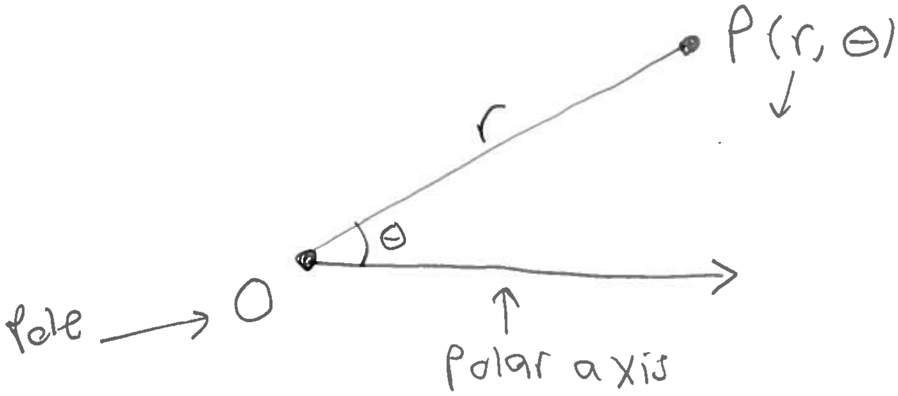
y: how many steps up/down from the origin

Motivation 2: Think of a boat at the origin and want to reach an island. How would you describe the path



e.g., 5 km in the directi
of NE = 'North East'

So we introduce a new coordinate system called the Polar Coordinat



So we describe any point P by the distance from O and the angle

So $(\overset{\text{radial}}{r}, \overset{\text{angular}}{\theta})$ is called the <u>polar coordinates</u> while (x, y) is called the <u>Cartesian coordinates</u>

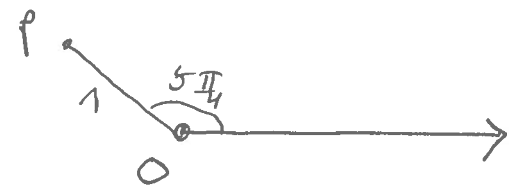
<u>Example 1</u>: Plot the following points in polar coordinate system.

(a) $\left(1, \frac{3\pi}{4}\right)$ (b) $(2, 3\pi)$, (c) $\left(2, -\frac{2\pi}{3}\right)$, (d) $\left(-3, \frac{3\pi}{4}\right)$

<u>Solution</u>:

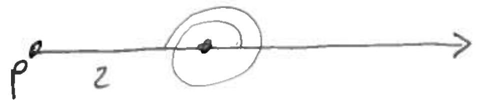
(a) $r = 1$

$\theta = \frac{5\pi}{4} = 135°$



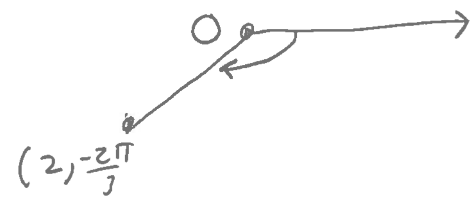
(b) $r = 2$

$\theta = 3\pi = 420$



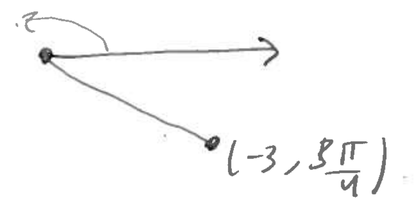
(c) $r = 2$

$\theta = \frac{-2\pi}{3} = -120°$



(d) $r = -3$

$\theta = \frac{3\pi}{4}$

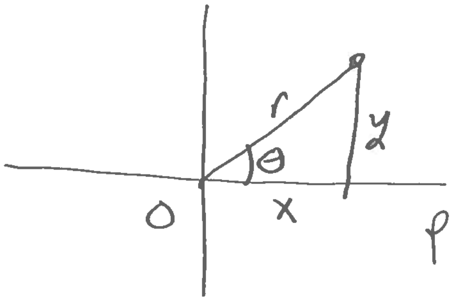


<u>Note</u>:

Points in the polar coordinates system differ by a complete rotation

$$(r, \theta) = (r, \theta + 2n\pi) = (-r, \theta + (2n+1)\pi).$$

2- Converting polar coordinate into cartesian coordinates



C → P $\quad x^2+y^2=r^2$, $\tan\theta=\frac{y}{x}$

P → C $\quad x=r\cos\theta$, $y=r\sin\theta$.

Example 2: Convert $(2,\frac{\pi}{3})$ into cartesian coordinate

$x=r\cos\theta$, $y=r\sin\theta$

$x=2\cos\frac{\pi}{3}$, $y=2\sin\frac{\pi}{3}$

$x=2\cdot\frac{1}{2}=1$, $y=\frac{2\sqrt{3}}{2}=\sqrt{3} \rightsquigarrow (x,y)=(1,\sqrt{3})$.

Example 3: Convert $(1,-1)$ from cartesian into polar coordinate

$r^2=x^2+y^2$, $\tan\theta=\frac{x}{y}$

$=(1)^2+(-1)^2$, $\tan\theta=\frac{-1}{1}$

$=1+1$, $\tan\theta=-1$

$r^2=2 \rightsquigarrow \boxed{r=\sqrt{2}}$

$\theta=\tan^{-1}(-1)$

$\boxed{\theta=-\frac{\pi}{4}}$

$(\sqrt{2},\frac{-\pi}{4})$

Exercise 1: Convert each of the following into the respected coordinate.

(a) $(2,\frac{3\pi}{4})$ $\qquad$ (c) $(-2,3)$

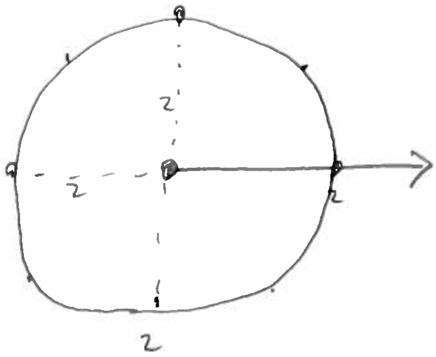
(b) $(1,0)$

3- Curve in polar coordinate

we will get a function $r = f(\theta)$ and we want to draw it

Example 3: sketch $r=2$.

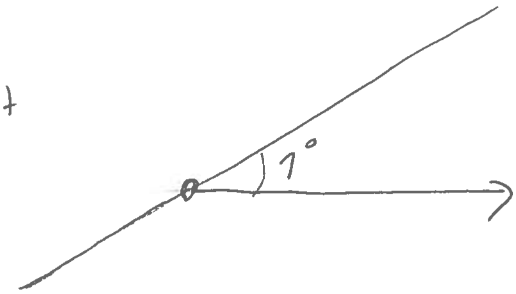
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$r=r(\theta)$	2	2	2	2	2



Since $x^2+y^2 = r = 2 \leftarrow$ Circle!

Example 4: sketch $\theta = 7$

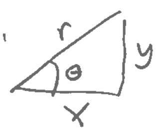
Now r is not in the equation! so it can be any number.



Example 5: sketch $r = 2\cos\theta$ and find the cartesian coordin

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$
r	2	$\sqrt{3}$	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2	

$\sqrt{x^2+y^2} = 2\cos\theta$,

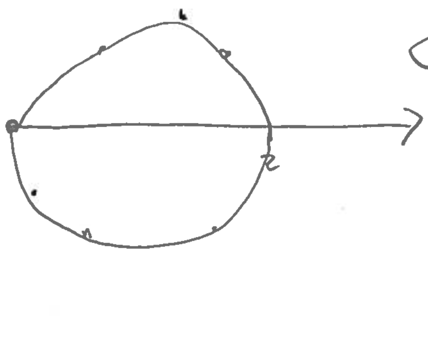


$\sqrt{x^2+y^2} = 2\,\frac{x}{r}$

$r^2 = 2x \to x^2+y^2 = 2x$

$x^2 - 2x + y^2 = 0$

$(x-1)^2 + y^2 = 1$.



Circle.

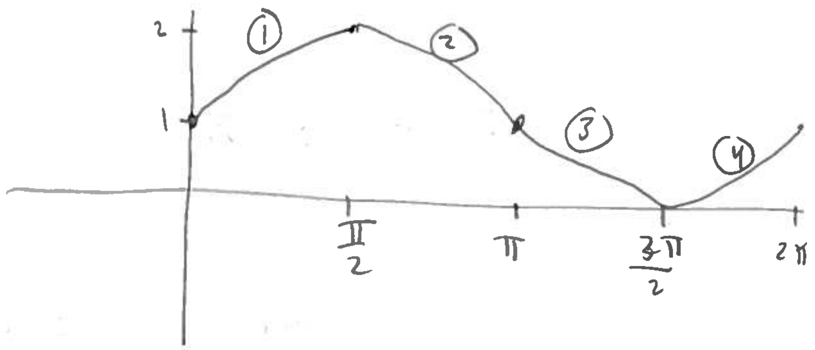
Exercise: Sketch the Graph and find the rectangular system of each of the following.

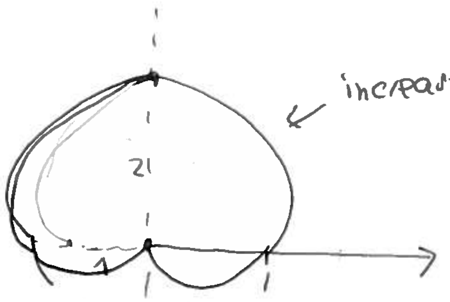
(a) $r = 12$ (b) $r = 6\theta$ (c) $r\sin\theta = 10$ (d) $r = 6\sin\theta$

Example 6: Sketch the curve $r = 1 + \sin\theta$.

(we can plot some points and draw the curve as before)

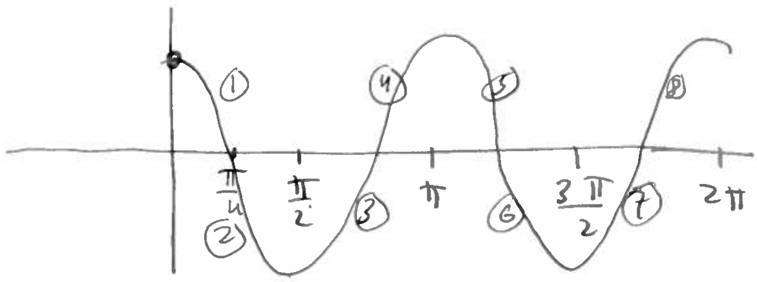
Step 1: Graph $y = 1 + \sin x$

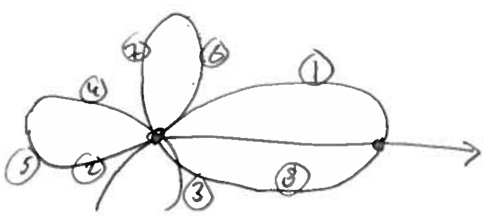


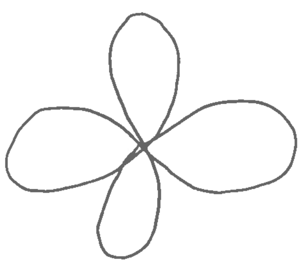


"cardioid"

Example 7: Sketch the graph $r = \cos 2\theta$





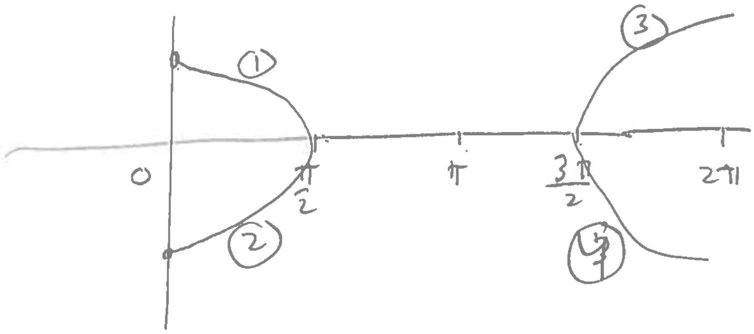


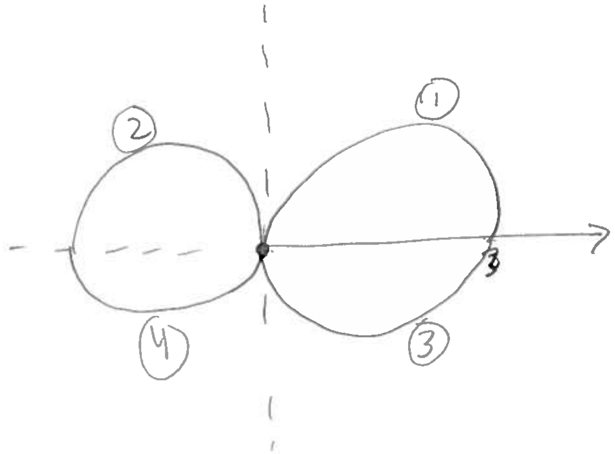
"four-leaved Flower"

Exercise 8: sketch $r = 3\sin 2\theta$

Example 8: Sketch $r^2 = 9\cos\theta$

$r = \pm 3\sqrt{\cos\theta}$





"lemniscate"