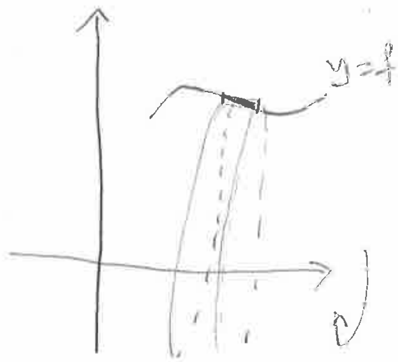
§6.4 - Area of a Surface of Revolution



Area of the part $= 2\pi f(x)\ \cancel{\text{length}}\ dx$

$$= 2\pi f(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

Surface area $= 2\pi\int_a^b f(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$

Example 1: Find the surface area by rotating around X-axis.

(a) $y = x^3, \quad 0 \le x \le 2$

Surface area $= 2\pi\int_0^2 f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi\int_0^2 x^3\sqrt{1+9x^4}\,dx$

$= \cdots$

(b) $y = \sin \pi x \quad , 0 \le x \le 1$

$y' = \pi\cos\pi x$

$S = 2\pi\int_0^1 \sin\pi x\sqrt{1+(\pi\cos\pi x)}\,dx$

(c) $x = 1+2y^2 \quad , 1 \le y \le 2$

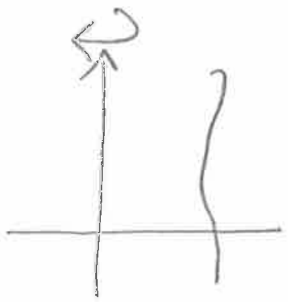
$x' = 4y \quad S = 2\pi\int_1^2 (1+2y^2)\sqrt{1+(4y)^2}\,dy$

(d) $y = \sqrt{1+e^x}$, $0 \leq x \leq 1$

$$y' = \frac{e^x}{2\sqrt{1+e^x}}$$

$$S = \int_0^1 \sqrt{1+e^x}\,\sqrt{1+\frac{e^x}{2\sqrt{1+e^x}}}\,dx$$

Rotating around y-axis



$$S = 2\pi \int_c^d x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Example 2: Find the surface of revolution about y-axis.

(a) $y = 1 - x^2$, $0 \leq x \leq 1$

$$S = \int_0^1 x\sqrt{1+[y']^2}\,dx = 2\pi\int_0^1 x\sqrt{1+[-2x]^2}\,dx$$

$$= 2\pi\int_0^1 x\sqrt{1+4x^2}\,dx$$

(b) $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $1 \leq x \leq 2$

$$S = \int_1^2 2\pi x\sqrt{1+\left[\tfrac{1}{2}x - \tfrac{1}{2x}\right]^2}\,dx$$