

University of Bahrain  
College of Science  
Department of Mathematics  
Final Exam, Math 102  
Second Semester, 2012/2013

Name:

ID-Number:

Section:

|               |  |
|---------------|--|
| Serial Number |  |
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- 1) Write your name and numbers above.
  - 2) This exam consists of 6 questions.
  - 3) Show your work clearly and in details.
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|      | Ques. 1 | Ques. 2 | Ques. 3 | Ques. 4 | Ques. 5 | Ques. 6 | Out of 40 |
|------|---------|---------|---------|---------|---------|---------|-----------|
| Mark |         |         |         |         |         |         |           |

Q.1 (3+3=6 marks) Evaluate the following integrals:

a)  $\int \frac{1}{x(x+1)} dx$

b)  $\int \frac{dx}{x^2 \sqrt{x^2 - 1}}$

Q.2 (3+3=6 marks)

(a) Find the volume of the solid generated by revolving about the  $x$ -axis the region between  $y = 2x$  and  $y = x$  for  $0 \leq x \leq 1$ .

b) Find the area of the region below  $f(x) = \frac{1}{x^5}$  and above the  $x$ -axis on the interval  $[2, \infty)$ .

Q.3 (3+3=6 marks)

a) Find the following limit (if exist):  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + 1}{x + 1} \right)^{\frac{1}{x}}$

b) Use the table 

|   |
|---|
| $\frac{d}{dx}(\sin^{-1} u(x)) = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ |
| $\frac{d}{dx}(\tanh^{-1} u(x)) = \frac{1}{1-u^2} \frac{du}{dx}$       |

 to find the derivative of:

1)  $f(x) = x \sin^{-1}(\ln x)$

2)  $f(x) = \cosh^3(\tanh^{-1} x)$

Q.4 (3+3+3=9 marks)

Determine whether each of the following series converges or diverges:

(a)  $\sum_{n=1}^{\infty} \frac{(n+2)!}{(2n)!}$

(b)  $\sum_{n=1}^{\infty} \frac{\sin^3 n}{n^2}$

(c)  $\sum_{n=1}^{\infty} \frac{2^n}{n^n}$

Q.5 (7 marks)

Find the radius and interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3^{n+1}}$ .

Q.6 (3+3=6 marks)

a) Find the Maclaurin series of  $f(x) = \sinh x$ .

b) Approximate  $\int_0^1 e^{x^2} dx$  using the first three nonzero terms of the Maclaurin series of  $e^{x^2}$ .