University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 10: Irreducible and Prime elements
Due on May 3, 2018

Name: _____

1. In the quadratic ring $\mathbb{Z}[\sqrt{D}] := \{a + b\sqrt{D} \mid a, b \in \mathbb{Z}\}$, define the norm of element $x = a + b\sqrt{D}$ as
$$N(x) := |a^2 - b^2 D|$$
Prove the following properties:
    1. $N(x) = 0$ if and only if $x = 0$
    2. $N(x) = 1$ if and only if x is a unit in $\mathbb{Z}[\sqrt{D}]$.
    3. $(3 + 2\sqrt{2})^n$ is a unit in $\mathbb{Z}[\sqrt{2}]$

2. Show that $1 - i$, 3 are irreducible elements in $\mathbb{Z}[i]$ but not 2 and 5.

3. Prove 7 is irreducible in $\mathbb{Z}[\sqrt{6}]$ even though $N(7)$ is **not** prime.

4. Show that $2, 1+\sqrt{5}$ are irreducible elements but not prime elements in $\mathbb{Z}[\sqrt{5}]$.

5. In a PID, show that (a) is a maximal ideal if and only if a is an irreducible element. (Hint: Mimic the proof that was done for $k[X]$)