

University of Bahrain  
Department of Mathematics  
MATHS312: Abstract Algebra II  
Spring 2018  
Dr. Abdulla Eid



**Homework 11: PID, UFD, and ED**  
**Due on May 10, 2018**

Name: \_\_\_\_\_

1. Show that  $f(X) = 3X^2 + 4X + 3 \in \mathbb{Z}_5[X]$  can be factored into  $(3X + 2)(X + 4)$  and  $(4X + 1)(2X + 3)$ . Does this contradict the fact that  $\mathbb{Z}_5[X]$  is a UFD?

2. Show that  $2, 1 + \sqrt{3}i, 1 - \sqrt{3}i$  are irreducible elements in  $\mathbb{Z}[\sqrt{3}i]$  and that all are **not** associate. Conclude that  $\mathbb{Z}[\sqrt{3}i]$  is not an UFD.

3. Let  $R$  be a Euclidean domain with distance  $d$ . Show that  $u$  is a unit in  $R$  if and only if  $d(u) = 1$ .

4. Find the quotient and remainder of dividing  $3 - 4i$  by  $2 + 5i$ .

5. Find the  $g := \gcd(2 + 11i, 7 + 2i)$  and then find  $x, y$  such that  $g = x \cdot (2 + 11i) + y \cdot (7 + 2i)$ .