University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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# Homework 12: Extension Field
## Due on May 17, 2018

Name: _____

1. Use the proof of Kronecker's Theorem to find an extension field of $\mathbb{Z}_2$ that contains a zero for $f(X) = X^5 + X^4 + 1$. Is that extension a splitting field for $f(X)$?

2. Find the splitting field for the following polynomials along with the elements description.

   1. $f(X) = X^4 + X^2 + 1 = (X^2 + X + 1)(X^2 - X + 1) \in \mathbb{Q}[X]$.

   2. $f(X) = X^2 + X + 1 \in \mathbb{Z}_5[Z]$.

3. Describe the elements in each of the following fields.

   1. $\mathbb{Q}(\sqrt{2}, i)$.

   2. $\mathbb{Q}(\sqrt{2}, \sqrt{3}, i)$.

   3. $\mathbb{Q}(\sqrt{2} + \sqrt{-2})$.

   4. $\mathbb{Q}(\sqrt[n]{a}, \omega)$, where $\omega = e^{\frac{2\pi i}{n}}$.

4. Let $f(X) = 2X + 1 \in \mathbb{Z}_4[X]$. Is there a field extension of $\mathbb{Z}_4$ which could have a zero for $f(X)$. Does that contradict the conclusion of Kronecker's theorem?

   (Hine: if $\alpha$ is root for $f(X)$, what would be $2f(\alpha)$?)