University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 13: Algebraic and Finite Extensions
Due on May 24, 2018

Name: _____

1. Show that the following numbers are algebraic over $\mathbb{Q}$.
   1. $\sqrt{5}+\sqrt{2}i$

   2. $\sqrt{5}+\sqrt[3]{3}$

3. $\sqrt{\sqrt[3]{2}+i}$.

4. $\sqrt{\frac{1}{2}+\sqrt{5}}$

2. Find the degree and the basis for the following extionsion fields.

   1. $\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}i)$ over $\mathbb{Q}$.

2. $\mathbb{Q}(\sqrt{8})$ over $\mathbb{Q}(\sqrt{2})$.

3. $\mathbb{Q}(\sqrt{5}+\sqrt{2})$ over $\mathbb{Q}(\sqrt{5})$.

4. $\mathbb{Q}(\sqrt{3},\sqrt{5},\sqrt{7})$ over $\mathbb{Q}$.

3. Show that $\mathbb{Q}(\sqrt{3}, \sqrt{7}) = \mathbb{Q}(\sqrt{3} + \sqrt{7})$ and use it to generalize it to the case $\mathbb{Q}(\sqrt{a}, \sqrt{b}) = \mathbb{Q}(\sqrt{a} + \sqrt{b})$ is $\gcd(a, b) = 1$.

4. Show that $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}) = \mathbb{Q}(\sqrt[6]{2})$