

University of Bahrain  
Department of Mathematics  
MATHS312: Abstract Algebra II  
Spring 2018  
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## Homework 14: Galois Groups Due on May 24, 2018

Name: \_\_\_\_\_

1. Find the Galois group for the following extension fields. (Refer to Homework 13, Question 2)

1.  $\mathbb{Q}(\sqrt{6}, i)$  over  $\mathbb{Q}$ .

2.  $\mathbb{Q}(\sqrt[3]{5}, \sqrt{5}i)$  over  $\mathbb{Q}$ .

3.  $\mathbb{Q}(\sqrt{5} + \sqrt{2})$  over  $\mathbb{Q}(\sqrt{5})$ .

4.  $\mathbb{Q}(\sqrt{3}, \sqrt{5}, \sqrt{7})$  over  $\mathbb{Q}$ .

5.  $X^5 - 12X^2 + 2$  over  $\mathbb{Q}$ .

2. Let  $F := \mathbb{Q}(\sqrt[3]{3}, \omega)$ , where  $\omega$  is a primitive root of unity.

(a) Find exactly the value of  $\omega$  as a complex number in the standard form  $x + iy$ .

(b) Find a polynomial of minimal degree that has  $\omega$  as a zero.

(c) Find a basis for the extension field  $F$  over  $\mathbb{Q}$ . What is  $[F : \mathbb{Q}]$ ?

(d) Find the Galois group  $G$  of the extension  $F$  over  $\mathbb{Q}$  and describe the action of each automorphism.

(e) Given the fact that  $G$  has total of six subgroups, 1 of them of size 1, 3 of them are cyclic of size 2, 1 of them is cyclic of size 3, and 1 of size 6. Exhibit the Galois corresponding between the subgroups and the subfields.