University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 2: Subrings Examples
Due on March 1, 2018
Hand in all problems

Name: _____

1. Show that $\mathbb{Z}[\sqrt{2}]$ is a subring of $\mathbb{R}$.

2. Show that $\mathbb{Z}[\sqrt{2}, \sqrt{3}] := \{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} \mid a, b, c, d \in \mathbb{Z}\}$ is a subring of $\mathbb{R}$.

3. Show that $\mathbb{Q}(\sqrt{D})$ is a subfield of $\mathbb{C}$ for any integer D.

4. (Ring of integers localized at p). For any prime p, define

$$\mathbb{Z}_{(p)} := \left\{ \frac{a}{b} \mid \gcd(b, p) = 1 \right\}$$

Show that $\mathbb{Z}_{(p)}$ is a subring of $\mathbb{Q}$.