University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 4: Zero divisors, units, and Integral domain
Due on March 22, 2018
Hand in problems 1–7

Name: ______________________________

1. Describe the units of $\mathbb{Z}_4[i]$.

2. Find all zero divisors in $\mathbb{Z}_{30}$ and $\mathbb{Z}_5[i]$.

3. Find the units in the ring in Homework 1, Exercise 1.

4. Let S be a subring of R.

   (i) Show by example that the unity 1_S need not to be the same as the unity of 1_R.

   (ii) If R is an integral domain, then $1_R = 1_S$.

5. Is $\mathbb{Z}[\sqrt{D}]$ an integral domain?

6. Let R be a finite commutative ring with unity. Prove that every nonzero element of R is either a unit or a zero–divisor.

   (Hint: The proof of Wedderburn's theorem)

7. (Important)

   Let R be an integral domain.

   (i) If char(R)=2, then $(x+y)^2 = x^2 + y^2$ and that $(x+y)^{2^n} = x^{2^n} + y^{2^n}$.

   (i) If char(R)=p, then $(x+y)^p = x^p + y^p$ and that $(x+y)^{p^n} = x^{p^n} + y^{p^n}$. (Hint: Use the fact that $p \mid \binom{p}{k}$)

8. Let u be a unit in a commutative ring with unity and $b^2 = 0$. Show that $u + b$ is a unit.