University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 6: Quotient ring, prime and maximal ideals
Due on April 12, 2018

Name: _____

1. Let R be a commutative ring with unity and I is an ideal of R. Prove that $(R/I, +, \cdot)$ is a commutative ring with unity.

2. Let $R = \mathbb{Z}[i]/(3+i)$.

   1. What are the elements of R?
   2. What is $3 + i + I$?
   3. What is the relation between $-3 + I$ and $i + I$?
   4. What is $5 + 5i + I$ equal to?
   5. What does R looks like?

2. Let $R = \mathbb{Z}[i]/(3+i)$.

3. (a) Assume R/I is an integral domain. Prove that I is a prime ideal.

    (b) Assume I is a maximal ideal. Prove that R/I is a field.

4. Prove that $I = (2 + 2i)$ is not a prime ideal of $\mathbb{Z}[i]$. How many elements are in $\mathbb{Z}[i]/I$?

5. (Bonus) Prove that $M = \{a + bi \mid 3 \mid a, 3 \mid b\} \subseteq \mathbb{Z}[i]$ is a maximal ideal of $\mathbb{Z}[i]$.