University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 7: Ring Homomorphism
Due on April 12, 2018

Name: ____

1. Let $\varphi : R \to S$ and $B \subseteq S$ is a maximal ideal. Show that $\varphi^{-1}(B)$ is a maximal ideal.

2. Let $R = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix}, a, b \in \mathbb{Z} \right\}$. Define a map

$$\varphi : R \to \mathbb{Z}$$
$$\begin{pmatrix} a & b \\ b & a \end{pmatrix} \mapsto a - b$$

1. Show that R is a ring homomorphism.
2. Find $\ker(\varphi)$.
3. Show $R/\ker(\varphi) \simeq \mathbb{Z}$.
4. If $\ker(\varphi)$ a prime ideal? maximal ideal?

3. (Frobenius map) Let char(R)=p, show that the map

$$\begin{aligned} \varphi : R &\to R \\ x &\mapsto x^p \end{aligned}$$

is a ring homomorphism.

4. Show that a homomorphism from a field onto a nontrivial ring is an isomorphism. (Hint: Use the first isomorphism theorem)

5. Show that $\mathbb{Z}[\sqrt{2}]$ is not ring isomorphic to $\mathbb{Z}[\sqrt{5}]$.