University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 8: Polynomials Part 1
Due on April 19, 2018

Name: _____

1. Complete the proof of the division algorithm in the polynomial ring over a field.

2. Find the quotient and reminder when dividing $f(X)$ by $g(X)$:

(a) $f(X) = 6X^4 + 3X^3 + X + 2, g(X) = 5X^2 + 3X + 1$ over $\mathbb{Z}_7$.

(b) $f(X) = \frac{7}{3}X^3 - X + \frac{17}{2}, g(X) = \frac{5}{8}X - \frac{2}{11}$ over $\mathbb{Q}$.

3. Proof Gauss lemma, i.e., if $f(X) \in \mathbb{Z}[X]$ is irreducible over $\mathbb{Q}$, then $f(X)$ is irreducible over $\mathbb{Z}$.
   (Hint: Mimic the example given in the class)

4. Show that every ideal I in $k[X]$ is principal.
   (Hint: Use the division algorithm and mimic the same proof for $\mathbb{Z}$)