University of Bahrain
Department of Mathematics
MATHS312: Abstract Algebra II
Spring 2018
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Homework 9: Polynomials Part 2
Due on April 26, 2018

Name: ______________________________

1. show that $f(X) = X^2 + 3X + 2$ has four zeros over $\mathbb{Z}_6$. Does that contradict the fact that every polynomial of degree n must have at most n zeros? Explain

2. List all polynomials of degree 2 over $\mathbb{Z}_2$.

3. (a) Show that $f(X) = x^2 + x + 4$ is irreducible over $\mathbb{Z}_{11}$.

(b) Let $f(x) = x^3 + 6$. Write f(x) as a product of irreducible polynomials over $\mathbb{Z}_7$.

(c) Let $f(x) = X^3 + X^2 + X + 1$. Write f(x) as a product of irreducible polynomials over $\mathbb{Z}_2$.

(d) Find all the zeros and their multiplicities of $X^5 + 4X^4 + 4X^3 - X^2 - 4X + 1$ over $\mathbb{Z}_5$.

4. Determine which of the polynomials below is (are) irreducible over $\mathbb{Q}$.
   a. $X^5 + 9X^4 + 12X^2 + 6$
   b. $X^4 + X + 1$
   c. $X^4 + 3X^2 + 3$
   d. $X^5 + 5X^2 + 1$
   e. $(5/2)X^5 + (9/2)X^4 + 15X^3 + (3/7)X^2 + 6X + 3/14$

5. Consider $f(X) = X^4 + 1$.

(a) Show that $f(X)$ is irreducible over $\mathbb{Z}$ and $\mathbb{Q}$.

(b) Show that $f(X)$ is irreducible over $\mathbb{R}$.

(c) Show that $f(X)$ is reducible over $\mathbb{Z}_p$ for every prime p.

(d) Explain why the converse of the mod p irreducibility test does not work.