TC2MA324 Quiz 3 March 25, 2015

University of Bahrain
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TC2MA324: History of Mathematics
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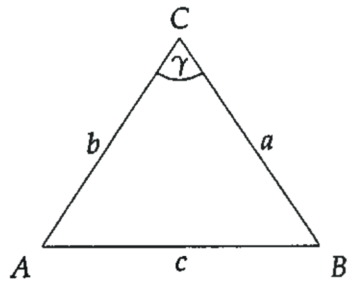
Quiz 3

Name: Solution

1. Consider the following triangle with sides of length a, b, c.



1 pt (a) State AlKashi theorem (law of cosine) for the triangle above.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

2 pt (b) If the sides of the triangles are 1,1, and 2. Can you find all the angles of the triangle? how?

$$2^2 = 1^2 + 1^2 - 2(1)(1)\cos\gamma$$

Isosceles Triangle

$$4 = 2 - 2\cos\gamma$$

$$2 = 1 - \cos\gamma \Rightarrow \cos\gamma = 1 - 2 = -1 \Rightarrow \gamma = \pi$$

which cannot be the case!

1 pt (c) Can you use AlKashi theorem to prove Pythagorean theorem? Why?

No!, because in the proof of AlKashi theorem, we use the Pythagorean theorem.

1.5 pt 2. (a) State Wilson's theorem as stated by Ibn Al–Haytham.

$n>$ is prime if and only if

n divides $(n-1)!+1$

2 pt (b) Apply it to verify that 8 is a composite number.

$(8-1)!+1 = 7!+1 = 5040+1 = 5041$

and $5041 = 8 \times 630 + 1$, so 5041 cannot be divisible by 8

reminder of 1

So 8 is not prime.

1.5 pt 3. (a) Define what does it mean that two numbers a and b are amicable numbers (friendly numbers)?

The sum of the divisor of a (without a) is equal to the sum of the divisor of b (without b), i.e.,

$$\sigma(a) = \sum_{\substack{d \mid a \\ d \neq a}} d = \sigma(b) = a+b.$$

3 pt

(b) Ibn Qurra theorem states that if

$$p := 3 \cdot 2^{n-1} - 1$$
$$q := 3 \cdot 2^{n} - 1$$
$$r := 9 \cdot 2^{2n-1} - 1$$

where $n > 1$, p, q, r are all prime number, then

$$a := 2^n \cdot p \cdot q, \qquad b := 2^n \cdot r$$

are amicable numbers. Find two such pairs of amicable numbers.

$n = 2 \Rightarrow p = 3 \cdot 2 - 1 = 5$ (prime)
$q = 3 \cdot 2^2 - 1 = 11$ (prime)
$r = 9 \cdot 2^3 - 1 = 71$ (prime)

$a = 4 \cdot 5 \cdot 11 = \boxed{220}$
$b = 4 \cdot 71 = \boxed{284}$

$n = 3 \Rightarrow p = 11$, $q = 23$, but $r = 287$ (composite) $= 7 \cdot 41$

$n = 4 \Rightarrow p = 23$, $q = 47$ $r = 1151 \leftarrow$ prime

$\boxed{a = 17296 \qquad , b = 18416}$

4. One row of pascal's triangle containing the following coefficients:

1 pt

1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

Use the idea of Ibn AlKhayyam to produce the row immediately following this row in Pascal's triangle.

1 14 91 364 1001 2002 3003 3432 2002 1001 364 91
14 1