TC2MA324 Quiz 4 April 8, 2015

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TC2MA324: History of Mathematics
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Quiz 4

Name: Solution

1. (5 points) (a) Find the number of possible positive and negative roots of the following equation using the rule of signs technique by Descartes.

$$x^4 + x^3 - 3x^2 + 6x - 1 = 0.$$

Positive roots

3 changes, so the possible number of positive roots is

3 or 1

negative roots

$f(-x) = x^4 - x^3 - 3x^2 - 6x - 1$

only one change in the sign so we have only one possibility of negative roots.

(b) Using Newton–Raphson method, approximate a positive root of the above equation using initial value of $x_0 = 2$.

$$f'(x) = 4x^3 + 3x^2 - 6x + 6$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 + x_n^3 - 3x_n^2 + 6x_n - 1}{4x_n^3 + 3x_n^2 - 6x_n + 6}$$

$x_0 = 2$

$x_1 = 1.394736842$

$x_2 = 0.8339904247$

$x_3 = 0.1802 \cdot 4893$ ... $\bar{x} \approx$

2. (4 points) Consider the function $y = x \sin x$.

(a) Find the derivative of y using Leibniz rule.

$$y' = x(\sin x)' + (x)' \sin x$$

$$\boxed{y' = x\cos x + \sin x}$$

(b) Using Newton–Raphson method, approximate a root of the equation

$$x \sin x = \frac{\pi}{2} \rightarrow x \sin x - \frac{\pi}{2} = 0$$

using $x_0 = 2$. (*Set your calculator to work with radian*)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n \sin x_n - \frac{\pi}{2}}{x_n \cos x_n + \sin x_n}$$

$x_0 = 2$

$x_1 = -1.298$

$x_2 = -1.532740613$

$x \approx -1.532740615$

3. (4 points) (a) Define the Euler–phi function $\phi(n)$.

$$\phi(n) = \#\{ m \mid m < n \text{ and } \gcd(m,n) = 1 \}$$

(b) Find the Euler–phi function of the following numbers. (*Show your work*)

(1) 17 (2) 33 (3)87 (4)26 (5) 105 (6) 29400

(1) $\phi(17) = 17 - 1 = 16$ --- 17 is prime

(2) $\phi(33) = \phi(3 \cdot 11) = \phi(3) \cdot \phi(11) = (3-1)(11-1) = 2 \cdot 10 = 20$

(3) $\phi(87) = \phi(29 \cdot 3) = \ldots = 28 \cdot 2 = 56$

(4) $\phi(26) = \phi(2 \cdot 13) = \phi(2)\,\phi(13) = (2-1)(13-1) = 12$

(5) $\phi(105) = \phi(3 \cdot 5 \cdot 7) = (3-1)(5-1)(7-1) = 48$

(6) $\phi(29400) = \phi(2^3 \cdot 5^2 \cdot 3 \cdot 7^2) = \phi(2^3)\,\phi(5^2)\,\phi(3)\,\phi(7^2)$

$= (2^3 - 2^2)(5^2 - 5)(3-1)(7^2 - 7) = 4 \times 20 \times 2 \times 42 = 6720$