TC2MA324 Quiz 5 April 15, 2015

University of Bahrain
Bahrain Teachers College
TC2MA324: History of Mathematics
Dr. Abdulla Eid
Spring 2015

Quiz ~~4~~ 5

Name: Solution

1. (3 points) Prove that the sum of the first $n+1$ terms of a geometric sequence is given by

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n = \begin{cases} \frac{a(r^{n+1}-1)}{r-1}, & \text{if } r \neq 1 \\ a(n+1), & \text{if } r = 1 \end{cases}$$

$$S_n = a + ar + ar^2 + \cdots + ar^{n-1} + ar^n \quad \text{—— (1)}$$

$$rS_n = ar + ar^2 + \cdots + ar^{n-1} + ar^n + ar^{n+1} \quad \text{—— (2)}$$

(2) − (1)

$$rS_n - S_n = ar^{n+1} - a \implies S_n(r-1) = a^{\cancel{n+1}}(r^{n+1}-1)$$

if $r \neq 1$, then $S_n = \dfrac{a(r^{n+1}-1)}{(r-1)}$

if $r=1$, then $S_n = \underbrace{a + a + a + \cdots + a}_{(n+1)\text{-time}} = a(n+1)$

2. (6 points) Find the sum of each of the following:

(a) $2+4+8+16+\cdots+1024$

2^{10} $a=2$, $r=2$, $n=9$

$$S_9 = \frac{a(r^{10}-1)}{(r-1)} = \frac{2(2^{10}-1)}{(2-1)} = 2^{11}-2 = 2046$$

(b) $\sum_{i=0}^{20} 5\left(\frac{2}{3^2}\right)^i$ $= \frac{5\left(\left(\frac{2}{3^2}\right)^{21}-1\right)}{\left(\frac{2}{3^2}-1\right)} = \frac{5\left(\left(\frac{2}{9}\right)^{21}-1\right)}{-7} \cdot 9$

(c) $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\ldots$ $= \frac{1}{1-\frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$

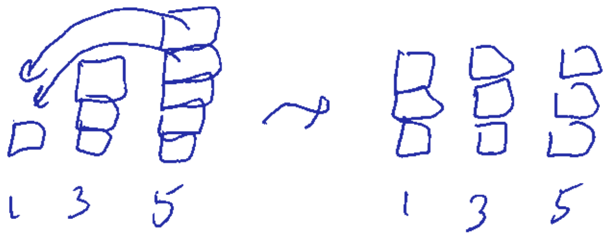
(d) $0.3+0.03+0.003+0.0003+\ldots$ $= \frac{0.3}{1-0.1} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{1}{3}$

3. (4 points) In this problem, you will need to guess a formula for the sum of the first n odd natural numbers using the geometry (similar to Gauss' original idea, but here try to create a square!)

(a) ($n = 2$) find the sum of $1 + 3$.

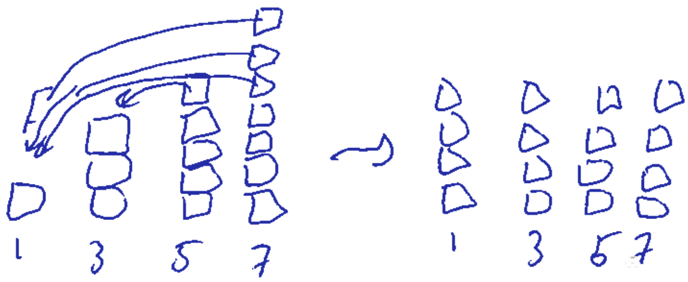
Total $= 4 = 2 \times 2 = 2^2$
$= n^2$

(b) ($n = 3$) find the sum of $1 + 3 + 5$.



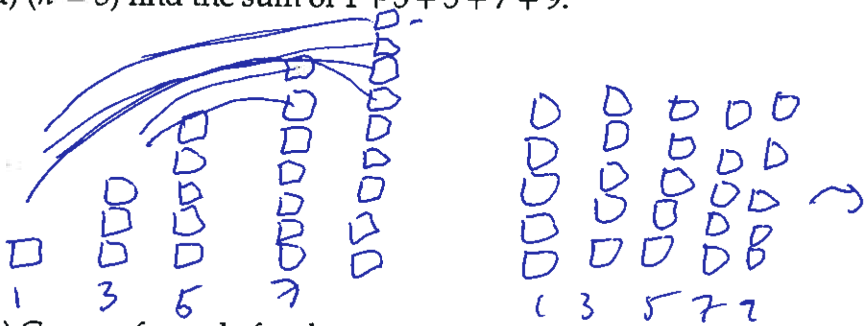
$\rightarrow$ Total $= 9 = 3 \times 3 = 3^2$

(c) ($n = 4$) find the sum of $1 + 3 + 5 + 7$.



$\rightarrow$ Total $= 16 = 4 \times 4 = 4^2$

(d) ($n = 5$) find the sum of $1 + 3 + 5 + 7 + 9$.



Total $= 25 = 5 \times$
$= 5^2$

(e) Guess a formula for the sum

$$1 + 3 + 5 + 7 + \cdots + (2n - 1)$$

(To prove, you will need to use the mathematical induction, but it is not required here.)

n^2