

University of Bahrain
 Bahrain Teachers College
 TC2MA324: History of Mathematics
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Quiz 7

Name: Solution

1. (6 points) Consider the following set of 17 preference ballot lists:

	7	5	4	1
1st choice	A	C	B	D
2nd choice	D	A	C	B
3rd choice	B	B	D	A
4th choice	C	D	A	C

Calculate the winner (if any) using:

- Condorcet's method. *No winner*
- Plurality Voting. *A*
- The Borda count. *A*
- Sequential pairwise voting with agenda A, B, D, C . *C*
- The Hare system. *A*
- The plurality runoff. *C*
- Use the above to show that the Hare system fails to satisfy the monotonicity property. *Suppose the last voter changed his ballot to*

	7	5	4	1
1st	A	C	B	A
2nd	D	A	C	B
3rd	B	B	D	D
4th	C	D	A	C

\rightarrow *C* is the winner.

2. (4 points) Consider the following sequence of 4 preference ballot lists:

	1	1	1	1		
1st choice	B	C	C	A	8	-1
2nd choice	A	A	D	D	4	-5
3rd choice	D	B	B	B	-4	-9
4th choice	C	D	A	C	-8	-13

1. Find the winner using Borda count.

A

2. Suppose we change the ways we assign points so that first place is worth 8 points, second place is worth 4 points, third place is worth -4 points and fourth place is worth -8 points. Find the winner using Borda count using these new numbers.

$$\begin{aligned}
 A &= 4 + 4 + (-8) + 8 = 8 \\
 B &= 8 + (-4) + (-4) + (-4) = -4 \\
 C &= -8 + 8 + 8 - 8 = 0 \\
 D &= -4 - 8 + 4 + 4 = -4
 \end{aligned}
 \left. \vphantom{\begin{aligned} A \\ B \\ C \\ D \end{aligned}} \right\} \text{A is the winner!}$$

3. Suppose we change the ways we assign points so that first place is worth -1 points, second place is worth -5 points, third place is worth -9 points and fourth place is worth -13 points. Find the winner using Borda count using these new numbers.

$$\begin{aligned}
 A &= -5 - 5 - 13 - 1 = -24 \\
 B &= -1 - 9 - 9 - 9 = -28 \\
 C &= -13 - 1 - 1 - 13 = -28 \\
 D &= -9 - 13 - 5 - 5 = -32
 \end{aligned}$$

So A is again a winner.

3. (3 points) 1. What are the desirable properties in the 2 candidates voting system?

- 1- Anonymity.
- 2- Neutrality.
- 3- monotonicity.
- 4- Decisiveness.

2. Explain why dictatorship method (one voter is a dictator and his vote yields the winner) doesn't satisfy all the conditions above.

He will choose the winner regardless of the others, so it will not satisfy the anonymity as if he exchange his ballot with other voter, the result will change.

3. State May's theorem.

Majority rule is the only rule that satisfies properties (1)-(4) for 2-candidates system.

① (3) $A = 32$, $C = 23$
 $B = 26$, $D = 21$