University of Bahrain
Bahrain Teachers College
TC2MA324: History of Mathematics
Dr. Abdulla Eid
Spring 2015

# Quiz 8

Name: Solution

1. (5 points) Use the following properties of the voting systems to match with their definition:

Always A Winner (AAW) Condorcet's Winner Criterion (CWC)
Monotonicity
Independent of Irrelevant Alternatives (IIA) Paerto Condition

(1) Monotonicity is the property that if some candidate $A$ is a winner and a new election is held in which the only ballot change made is for some voter who move $A$ higher on his ballot, then $A$ will remain a winner.

(2) IIA is the property that it is impossible for a candidate $A$ to move from non-winner state to a winner state unless at least one voter reverses the order of $A$.

(3) Always A winner is the property that there will be always a winner.

(4) Paerto Condition is the property that in every election in which every voter prefer $A$ over $B$, then $B$ shouldn't be among the winners.

(5) CWC is the property that the winner is the same winner if the Condorcet's voting system is used.

2. (3 points) Use the following preference ballot lists to show that Hare system, and plurality voting system don't satisfy independence of irrelevance alternatives (IIA).

| | 6 | 4 | 3 | 4 |
|---|---|---|---|---|
| 1st choice | A | B | C | D |
| 2nd choice | B | A | B | C |
| 3rd choice | C | C | A | B |
| ~~3rd~~ 4th choice | D | D | D | A |

Hare System:

Winner is B

If a new election, the last 4 voters reverse the order of D & C to get

| | 6 | 4 | 3 | 4 |
|---|---|---|---|---|
| | A | B | C | C |
| | B | A | B | D |
| | C | C | A | B |
| | D | D | D | A |

Then the winner is A!!

Plurality voting: Winner is A.

If the last 4 voter again reverse the orde of D & C. Then the winner is C.

3. (3 points) Find each of the following quantities:

1. 190 mod 11. $= 3$

2. 219 mod 11. $= 10$

3. 407 mod 11. $= 8$

4. $407^{22}$ mod 11. $= 8^{22} \bmod 11 = 0$

5. $190 \cdot 219$ mod 11. $= 3 \cdot 10 \bmod 11 = 30 \bmod 11$
$= 8$

6. $219^{120}$ mod 11.

$219^{120} \bmod 11 = 10^{120} \bmod 11 = (-1)^{120} \bmod 11$
$= 1 \bmod 11 = 1$

4. (2 points) State Arrow's impossibility theorem.

It is impossible to have a voting system of three or candidates that satisfy

(1) AAW

(2) IIA

(3) pareto condition.