Section 2.2
Special Functions

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MATHS 103: Mathematics for Business I

Example

(Constant Function) Consider the function

$$f : (-\infty, \infty) \to (-\infty, \infty)$$
$$x \mapsto 3$$

or simply by $f(x) = 3$

- f(1)=3. f(0)=3.
- f(-2)=3. f(-7)=3.

The output is always 3 (regardless of the input)

- Domain $= (-\infty, \infty)$.
- Co–domain$=(-\infty, \infty)$.
- Range$=\{3\}$.

Constant Function

Let c be any fixed number

Definition

The constant function at c is the function

$$f(x) = c$$

Question: What is a linear equation? quadratic equation?

Answer:

1. Linear equation: $ax + b = 0$.
2. Quadratic equation: $ax^2 + bx + c = 0$.

Question: What is a linear function? quadratic function?

Answer:

1. Linear function: $f(x) = ax + b$.
2. Quadratic function: $f(x) = ax^2 + bx + c$.

Definition

A **polynomial** function is a function of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where $a_n, a_{n-1}, \ldots, a_1, a_0$ are real numbers are called coefficients with $a_n \neq 0$ is called leading coefficient and n is an integer called the degree of $p(x)$.

Example

$f(x)$	Degree	Leading Coefficient
$3x+1$		
x^2+x+1		
$1+2x+2x^3+x^2$		
7		
$\frac{x^2+5x+10}{7}$		

Example

(Non–Polynomial Examples)

1. $f(x)=\frac{x^2+1}{x}$.
2. $f(x)=\sqrt{x}$.
3. $f(x)=3^x$.

Definition

A **rational** function is a quotient of two polynomials (polynomial over polynomial).

Example

1. $f(x) = \frac{x^2+1}{x}$.
2. $f(x) = \frac{x^2-2x}{x^4+x^7}$.
3. $f(x) = \frac{2x^3+1}{7x+2}$.
4. $f(x) = x^{-4} = \frac{1}{x^4}$.

Example

(Case–Defined Functions)

$$g(x) = \begin{cases} x - 1, & x \geq 3 \\ 3 - x^2, & x < 3 \end{cases}$$

- g(1)=$3 - (1)^2$=2.
- g(-2)=$3 - (-2)^2$=-1.
- g(6)=$6 - 1$=5.
- g(4)=$4 - 1$=3.
- g(3)=$3 - 1$=2.

Exercise

Consider the function

$$F(x) = \begin{cases} 2, & t > 1 \\ 0, & t = 1 \\ -1, & t < 1 \end{cases}$$

Find the following: $F(12), F(-\sqrt{3}), F(1), F(\frac{15}{2})$.

Exercise

Consider the function

$$f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Find the following: $f(1), f(-2), f(-7), f(7), f(-12022), f(0)$. Can you recognize this function?