University of Bahrain
College of Science
Department of Mathematics
Second Semester 2012/2013
Final Examination

Course: MATHS 253
Date: 15/06/2013
Time: 11:30-13:30

Name:

ID Number:

Serial Number: Section Number

PLEASE MAKE SURE YOUR COPY OF THIS EXAMINATION CONSISTS OF 8 PAGES AND 6 DIFFERENT QUESTIONS.

Question	Max. Marks	Marks obtained
1	16	
2	12	
3	12	
4	16	
5	12	
6	12	
Total	80	

Show all your work clearly

Question 1:[5+5+6 marks]

Prove each of the following

1) **Premises:** $s \Rightarrow p \wedge r, \neg p \ , \neg s \wedge \neg p \Leftrightarrow \neg q$
   Conclusion: $\neg q$

2) If $xz < yz$ and $z < 0$ then $x > y$. (Hint: Use contradiction)

3) 3 is a factor of $n^3 - n$ for $n = 0, 1, 2, \ldots$ (Hint: Use induction)

Question 2:[2 marks each]

Complete each of the following statements

1) The domain of definition of $f(x)=\dfrac{\sqrt{x-1}+4}{x^2+x-2}$ is.....

2) $(\forall x \in B)\ p(x)$ means $\forall x$ (.....)

3) For any statement P, $P \wedge \neg P$ is a .....and $P \vee \neg P$ is a .....

4) The negation of $\forall z\,(\exists \varepsilon > 0)\ 0 \le z < \varepsilon \Rightarrow z = 0$ is

.....

5) If $f:(0,\infty) \to IR$ is the function defined by $f(x)=x^2-3$ and $B=[1,6]$, then $f^{-1}(B)=$.....

6) $A \times B = \phi \Leftrightarrow$ .....

7) The power set $P(\{\phi, \{\phi\}\})=$.....

8) The contrapositive of $x>0$ and $y>0 \Rightarrow xy>0$ is .....

Question 3:[6+6 marks]

Prove each of the following using **the pick - a - point method**

1) $[(A \cap B) \cup (C - A)] \cup (B \cap C) \subset (A \cap B) \cup (C - A)$

2) $(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$.

Question 4:[6+6+4 marks]

1) Let $f:\mathbb{R}-\{1\}\to\mathbb{R}$ be the function defined by $f(x)=\dfrac{x+2}{x-1}$. Show that f is one to one.

2) Let $f:D\to C$ and $g:C\to A$ be two functions

a) Prove the statement (f onto and g onto $\Rightarrow$ gof onto)

b) Write the converse of the statement in part a), then decide if is true or false justifying your answer.

Question 5: [6+6marks]

Let $f : D \to C$ be a function. For any subsets A and B of D

1) Prove $f(A) - f(B) \subset f(A - B)$.

2) Prove that if f is one to one, then $f^{-1}(f(A)) \times A \subset A \times A$.

Question 6:[7+5 marks]

Define a relation $\mathcal{R}$ on the set $\mathbb{R} \times \mathbb{R}$ by $(x, y)\ \mathcal{R}\ (u, v)$ if and only if $x^2 + y^2 = u^2 + v^2$

1) Prove that $\mathcal{R}$ is an equivalence relation.

2) List the elements in each of the equivalence classes $[(0, 0)]$ and $[(1, 0)]$. Show your work.