University of Bahrain Department of Mathematics MATHS253: Set Theory Fall 2018 Dr. Abdulla Eid



## Homework 11: Collection of Sets

Name: \_

1. Let  $S_n = [n - 1, n + 3] \subset \mathbb{R}$  and  $I = \{1, 3, 4\}$ . Find  $S = \{S_i | i \in I\}, \bigcup_{i \in I} S_i$  and  $\bigcap_{i \in I} S_i$ . Draw your results in the real line.

2. Let  $\{A_i : i \in I\}$  be a collection of sets and let  $j \in I$ . Prove that

$$\bigcap_{i\in I} A_i \subseteq A_j \subseteq \bigcup_{i\in I} A_i$$

3. Let  $\{A_n : n \in \mathbb{N}\}$  be a collection of sets. Prove the following:

(a) If  $A_0 \subseteq A_1 \subseteq A_2 \subseteq \ldots$ , then  $\bigcap_n A_n = A_0$ 

(b) If  $A_0 \supseteq A_1 \supseteq A_2 \supseteq \dots$ , then  $\bigcup_n A_n = A_0$ 

4. Let  $A_n = \{x \in \mathbb{N} \mid x \ge n\}$ . Prove that  $\bigcup_{n \in \mathbb{N}} A_n = \mathbb{N}$  and  $\bigcap_{n \in \mathbb{N}} A_n = \emptyset$ .

## MATHS 253 Homework 11: Collection of Sets, Page 3 of ??

5. (a) Prove the Arichmedian property of the real number: For all x > 0, there exists positve integer n such that  $0 < x < \frac{1}{n}$ .

(b) Let 
$$C_n = \left[0, \frac{1}{n}\right)$$
  $(n \in \mathbb{N} - \{0\})$ . Show that  
$$\bigcap_{n=1}^{\infty} C_n = \{0\}$$

6. Let  $A_i$  ( $i \in I$ ) be a collection of sets and B be a set. Prove the following:

(a)  $B \cap (\bigcup_{i \in I} A_i) = \bigcup_{i \in I} (B \cap A_i)$ 

(b)  $B \cup (\bigcap_{i \in I} A_i) = \bigcap_{i \in I} (B \cup A_i)$ 

(c) 
$$\left(\bigcup_{i\in I} A_i\right)^c = \bigcap_{i\in I} A_i^c$$