University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
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Homework 15: Equivalence Relation, part 1
Due December 31, 2018

Name: ______________________________

1. Let $A := \{0, 1, 2, 3, 4\}$. Define a relation R in A as follows:

$$R = \{(0,0),(0,4),(1,1),(1,3),(2,2),(3,1),(3,3),(4,0),(4,4)\}$$

   (a) Show R is an equivalence relation.

   (b) Find all equivalence classes of R.

2. Let $A := \mathbb{Z}$. Define $\sim$ on A as follows

$$x \sim y :\iff 11x - 5y \text{ is even}$$

Show that $\sim$ is an equivalence relation and find $A/\sim$.

3. Let $A := \mathbb{Z}$. Define $\sim$ on A as follows

$$x \sim y :\iff x^3 = y^3$$

Show that $\sim$ is an equivalence relation and find $A/\sim$.

4. (a) Let $A := \mathbb{Z}$. Define $\sim$ on A as follows

$$x \sim y :\iff x + y \text{ is even}$$

Show that $\sim$ is an equivalence relation and find $A/\sim$.

(b) If we replace 'even' with 'odd' in Part (a), is it still an equivalence relation?

5. Let $A := \mathbb{R}^2$. Define $\sim$ on A as follows

$$(a, b) \sim (x, y) :\iff a^2 + b^2 = x^2 + y^2$$

Show that $\sim$ is an equivalence relation. Describe geometrically the equivalence classes $[(0,0)], [(0,1)], [(1,0)]$.

6. In this exercise, we will show that the set of rational numbers can be constructed from an equivalence relation.

   Let $A := \mathbb{Z} \times (\mathbb{Z} - \{0\})$. Define $\sim$ on A as follows

$$(a, b) \sim (c, d) :\iff ad - bc = 0$$

   Show that $\sim$ is an equivalence relation. Describe the equivalence classes $[(1,2)], [(0,1)], [(1,0)]$. Can you see the relation between $A/\sim$ and $\mathbb{Q}$?

7. Let $\sim$ be an equivalence relation on A, show that if $a \sim b$, $c \sim d$, $a \sim d$, then $b \sim c$.

8. (a) Let $A = \{1, 2, 3\}$. Define an equivalence relation $\sim$ on the power set of A as follows:

$$\text{For all sets } U, V \text{ in } P(A), U \sim V :\Longleftrightarrow |\, U \,|=|\, V \,|$$

Find the distinct equivalence classes.

(b) Let $A = \{1, 2, 3, \ldots, n\}$. Define the equivalence relation on A as in Part (a). Find all the distinct equivalence classes.

9. Suppose that A is non-empty set and $f : A \to A$ is a function. Define a relation $\sim$ on A as follows:
$$x \sim y :\iff f(x) = f(y)$$
Show that R is an equivalence relation. If f is injective, how many equivalence classes would be there?

10. Recall the following equivalence relation on the set of integers:
$$x \sim y :\iff x = y \text{ or } x = -y$$
Write the equivalence classes $[0], [1], [-1], [2], [6]$, and in general $[n]$, for any integer n.

11. Let d be a fixed integer. On the set of integers, define the following relation

$$x \equiv_d y :\iff x = y + kd, \text{ for some } k \in \mathbb{Z}$$

(a) Show that $\equiv_d$ is an equivalence relation.

(b) What is the equivalence class $[0], [1], [2]$.

(c) Let $d = 2$. Find all *distinct* equivalence classes.

(d) Let $d = 3$. Find all *distinct* equivalence classes.