University of Bahrain Department of Mathematics MATHS253: Set Theory Fall 2018 Dr. Abdulla Eid



## Homework 16: Partial Ordering Due December 31, 2018

Name: \_\_\_\_\_

1. Prove that the least upper bound of a poset  $(A, \preceq)$  is unique.

- 2. Consider the poset ({2,4,6,9,12,18,27,36,48,60,72},|).
  - (a) Draw the Hasse diagram for the above poset.

- (b) Find the maximal elements.
- (c) Find the minimal elements.
- (d) Is there a greatest element.
- (e) Is there a least element.
- (f) Find all upper bounds of  $\{2,9\}$ .
- (g) Find (if any) the least upper bound of  $\{2,9\}$ .

- (f) Find all lower bounds of  $\{60, 72\}$ .
- (g) Find (if any) the greatest lower bound of  $\{60, 72\}$ .
- 3. Given two posets  $(A_1, \preceq_1)$  and  $(A_2, \preceq_2)$ , define the **lexicographic ordering (dic-***tionary ordering)*  $\preceq$  on  $A_1 \times A_2$  as follows

$$(a_1, b_1) \preceq (a_2, b_2) : \iff a_1 \prec a_2 \text{ or if } a_1 = a_2, \text{ then } b_1 \prec b_2$$

(a) Consider the usual  $\leq$  relation on  $\mathbb{Z}$ , is it true that  $(3,5) \leq (4,8), (4,9) \leq (4,11), (1,2,3,5) \leq (1,2,4,3)$ ?

(b) Find the lexicographic ordering of the following string of English letters: *quack, quick, quicksilver, quicksand, quacking* 

- 4. Consider the poset  $(\mathcal{P}(\{1,2,3,4\},\subseteq))$ .
  - (a) Draw the Hasse diagram for the above poset.

- (b) Find the maximal elements.
- (c) Find the minimal elements.
- (d) Is there a greatest element.
- (e) Is there a least element.
- (f) Find all upper bounds of  $\{\{2\}, \{4\}\}$ .
- (g) Find (if any) the least upper bound of  $\{\{2\}, \{4\}\}$ .

(f) Find all lower bounds of  $\{\{1,3,4\},\{2,3,4\}\}$ .

- (g) Find (if any) the greatest lower bound of  $\{\{1,3,4\},\{2,3,4\}\}$ .
- 5. (a) Let (A, R) be a poset. Show that  $(A, R^{-1})$  is also a poset. The poset  $(A, R^{-1})$  is called the **dual** of (A, R)

(b) Find the dual poset of  $(\{1, 2, 3, 4\}, \leq)$  and  $(\mathbb{Z}, |)$