

University of Bahrain
Department of Mathematics
MATHS253: Set Theory
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Homework 4: Rules of Inferences

Name: _____

1. Show that we can conclude q from the following set of premises:

1. $r \rightarrow p \vee q$.
2. $\neg p$.
3. r .

2. Show that we can conclude $\neg p$ from the following set of premises:

1. $p \rightarrow \neg r$.
2. $\neg r \rightarrow q$.
3. $\neg q$.

3. Show that we can conclude q from the following set of premises:

1. $p \vee q$.
2. $p \vee q \rightarrow (\neg r \rightarrow q)$.
3. $\neg r$.

4. Consider the following scenario: "If I go to the mall, then I will spend my money". "If I don't spend my money, then I can buy a new phone". "I couldn't buy a new phone".

Can you conclude that "I went to the Mall". Why?!

5. You visited an island with two natives, truthteller who always say the truth and liars who never say the truth and you encounter the following:

A says: "Both of us are truthtellers".
 B says: " A is a liar".

What are A and B ?

6. Show that all the rules of inferences we encounter in class are valid by showing that the following are tautologies:
1. (Modus Ponens) $p \wedge (p \rightarrow q) \rightarrow q$.
 2. (Modus Tollens) $(p \rightarrow q) \wedge \neg q \rightarrow \neg p$.
 3. (Disjunctive Syllogism) $(p \vee q) \wedge \neg q \rightarrow p$.
 4. (Hypothetical Syllogism) $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$.

7. * You are about to leave for school in the morning and discover that you don't have your glasses, You know the following statements are true:
1. "If my glasses are on the kitchen table, then I saw them at breakfast".
 2. "I was reading the newspaper in the living room or I was reading the newspaper in the kitchen".
 3. "If I was reading the newspaper in the living room, then my glasses are on the coffee table".
 4. "I didn't see my glasses at the breakfast".
 5. "If I was reading my book in bed, then my glasses are on the bed table".
 6. "If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table".

Where are the glasses?

(Hint: Assign a symbol to each sentence and use the rules of inference.)

8. ** Show that the rules Modus Tollens and Disjunctive syllogism can be derived from the Modus Ponens rule, i.e., given the premises $p \rightarrow q$ and $\neg q$, conclude $\neg p$ **using only** Modus ponens. Similarly given the premises $p \vee q$ and $\neg q$, conclude p **using only** Modus Ponens.

(Hint: Write $p \rightarrow q$ into some logically equivalent proposition that you have seen in class.)