University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
Dr. Abdulla Eid

Homework 5: Direct and Indirect Proof
Due Date: November 1, 2018

Name: ______________________________

1. Show using direct proof that the sum of odd integers is even.
2. Show using indirect proof that for all integers n if $3m+2$ is even, then n is even.
3. Prove that n is even if and only if $7n+4$ is even.
4. Let a, b are real numbers, prove that $ab = 0$ if and only if $a = 0$ or $b = 0$.
5. Prove that is a and b are even integers, then $ab + bc$ is even for all integers b.
6. Let n be an integer. Prove that $2n^2 + n$ is odd if and only if $\cos\left(\frac{n\pi}{2}\right)$ is even.
7. Let a, b be integers. Prove that ab is odd if and only if $a^2 + b^2$ is even. (Hint: Use proof by cases)
8. **Definition:** We call two integers a and b are of the **same parity** if either both of them are even or both of them are odd.
   1. Prove that m and n are of the same parity if and only if $n + m$ is even.
   2. Prove that if $3x + 5y$ is even, then x and y are of the same parity.
   3. Prove that if ab and $a + b$ are of the same parity, then a and b are even.

Divisibility

Definition: Let a and b be two integers with $a \neq 0$. We say a **divides** b if there exists integer k such that $b = ak$.

Alternatively we say

a **is a divisor of** b
a **is a factor of** b
b **is a divisible by** a
b **is a multiple of** a

and we denote it by $a \mid b$ ("a divides b"). Prove the following statements:

1. $a \mid b$ if and only if $\dfrac{b}{a}$ is an integer.
2. $7 \mid 21$, $3 \mid 81$, and $8 \nmid 9$.
3. $a \mid a$, $a \mid 0$, and $1 \mid b$.
4. Let a, b are positive integers. Prove if $a \mid b$, then $a \leq b$.
5. (Transitivity of divisibility) If $a \mid b$ and $b \mid c$, then $a \mid c$.
6. (Anti-symmetry of divisibility) Let a, b be positive integers. Prove if $a \mid b$ and $b \mid a$, then $a = b$.
7. If a, b, c, x, y are natural numbers such that $a \mid b$ and $a \mid c$, then $a \mid (bx + cy)$.
8. (Snake-like integers) Let's say that an integer y is **Snake–like** if there is some integer k such that $y = (6k)^2 + 9$.
   (a) Give three examples and three non-examples of Snake–like integers.
   (b) Give three examples of Snake–like integers with more than 7 digits. Can you see why it is called Snake–like?
   (c) Given integer y, compute the negation of the statement,"y is Snake–like".
   (d) Show that every Snake–like integer is a multiple of 9.
   (e) Show that the statements, "n is Snake-like," and, "n is a multiple of nine," are not equivalent.