University of Bahrain
Department of Mathematics
MATHS253: Set Theory
Fall 2018
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Homework 7: Mathematical Induction
Due Date: November 15, 2018

Name: _____

1. Prove that for all natural numbers $n \geq 1$,

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Show for all natural numbers n:

$$1 + 3 + 3^2 + 3^3 + 3^4 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}$$

3. Show for all natural numbers n:

$$1 + 6 + 11 + 16 + \cdots + (5n - 4) = \frac{n(5n - 3)}{2}$$

4. * (Linear Algebra) Prove that for all natural numbers $n \geq 1$

$$\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^n = \begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$$

5. * (The strong principal of mathematical induction)

   In the principal of mathematical induction we had in class was the induction hypothesis consists of only assuming $P(k)$ and we tried to prove $P(k+1)$. In the strong principal, the induction hypothesis consists of not only $P(k)$, but also $P(a) \wedge P(a+1) \wedge \ldots P(k-1) \wedge P(k)$.

   Use the strong principal of mathematical induction to prove the following: Let $\{F_n\}$ denote the Fibonacci sequence defined recursively by

$$\begin{aligned} F_n &= F_{n-1} + F_{n-2}, \qquad n \geq 2 \\ F_0 &= 1, F_1 = 1 \end{aligned}$$

   1. Find the first 9 terms of the Fibonacci sequence.

   2. Prove using the strong principal of mathematical induction that

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

3. Prove using the strong principal of mathematical induction that

$$F_{n-1}F_{n+1} - F_n^2 = (-1)^n$$

6. *

   (a) Prove that x^n is even if and only if x is even for all natural number $n \geq 1$ and integers x.

(b) Use (a) to show that $\sqrt[n]{2}$ is irrational number.

(c) Use (a) to show that $\log_2 3$ is irrational.