Section 12.3 Elasticity of Demand

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MATHS 104: Mathematics for Business II

Recall From Economics Class!

• The demand function is a function between the quantity *q* and the price *p*. usually we write

$$p = f(q)$$

• Under normal circumstances, if the price increases, then the quantity should decreases.

Goal: To determine the effect (consumer's response) to any increase in price. i.e., how the quantity will change if the price changes?

Definition

The demand of elasticity is a means to *measure* how a change in the price of a product will affect the quantity demanded.

Three cases of elasticity

• The demand is elastic if the change in price **do** effect the quantity demanded.

For example, if we have a luxury product!

- 2 The demand has unit elasticity if the change in price will result in the same change in the quantity demanded. For example, meat product!
- The demand has inelastic if the change in price will not effect too much the quantity demanded. For example, electricity!

Question: How to measure these mathematically?

Point of elasticity of demand

We find the point of elasticity of demand which is

$$\eta := \frac{\text{change in demand}}{\text{change in price}}$$
$$\eta := \frac{\frac{p}{q}}{\frac{dp}{dp}}$$
$$\eta := \frac{\frac{f(q)}{q}}{f'(q)}$$

The point of elasticity of demand η is pronounced as "eta".

- When $|\eta| > 1$, demand is elastic.
- **2** When $|\eta| = 1$, demand has unit elastic.
- **③** When $|\eta| < 1$, demand is inelastic.

(Old Exam Question) For the demand equation $p = 100 + 0.02q - q^2$ determine whether demand is elastic, is inelastic, or has unit elasticity for q = 5.

Solution: First we find the derivative of p which is

Next we find η

$$p' = 0.02 - 2q$$
$$\eta = \frac{\frac{p}{q}}{p'}$$
$$\eta = \frac{\frac{100 + 0.02q - q^2}{q}}{0.02 - 2q}$$

Now set q = 5 to get that

$$\eta = -1.505
ightarrow |\mathit{eta}| = 1.505$$

Hence the demand is *elastic*.

Exercise

(Old Final Exam Question) For the demand equation $p = \sqrt{4000 - q^2}$ determine whether demand is elastic, is inelastic, or has unit elasticity for q = 20.



Exercise

For the demand equation $p = 250e^{\frac{-q}{50}}$ determine whether demand is elastic, is inelastic, or has unit elasticity for q = 50.

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Next

(Old Exam Question) For the demand equation $p = \frac{150}{q+50}$ determine whether demand is elastic, is inelastic, or has unit elasticity for q = 50.

Solution: First we find the derivative of p which is using the quotient rule

$$p' = \frac{(q+50)(0) - 150(1)}{(q+50)^2} = \frac{-150}{(q+50)^2}$$

we find η
$$\eta = \frac{\frac{p}{q}}{p'}$$

$$\eta = \frac{\frac{\frac{150}{q+50}}{\frac{-150}{(q+50)^2}}$$

Now set q = 50 to get that

$$\eta = -2 \rightarrow |eta| = 2$$

Hence the demand is *elastic*.

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(Old Final Exam Question) If $\eta = -1.5$, then the demand is?

Solution:

$$\eta = -1.5
ightarrow |\eta| = 1.5$$

Hence the demand is *elastic*.

(Old Final Exam Question) For what value (or values) of q do the demand equation q = 144 - 4p has unit elasticity?

Solution: First we find the derivative of p which can be written as

$$ho=rac{144-q}{4}
ightarrow p'=rac{-1}{4}$$

since we have unit elasticity, we find η and we make it equal to 1 so

$$|\eta| = 1$$

$$\eta = 1 \text{ or } \eta = -1$$

$$\frac{\frac{p}{q}}{p'} = 1 \text{ or } \frac{\frac{p}{q}}{p'} = -1$$

$$\frac{\frac{144-q}{4}}{\frac{-1}{4}} = 1 \text{ or } \frac{\frac{144-q}{4}}{\frac{-1}{4}} = -2$$

Continue...

$$\frac{144 - q}{-q} = 1 \text{ or } \frac{144 - q}{-q} = -1$$

144 - q = -q or 144 - q = q
q = 72