Section 13.6
Applied Maxima and Minima

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MATHS 104: Mathematics for Business II

Example

A company manufacturer sells q phones per week. The weekly demand is given by

$$p = 500 - 0.5q$$

Find the selling price that maximizes the revenue.

Solution:

We find the revenue function to be

$$r = qp$$

We find the derivatives first which are

$$\begin{aligned} r' &= p + qp' \\ r' &= 500 - .5q + q(.5) \\ r' &= 500 - .5q + q(-.5) \\ r' &= 500 - q \\ r'' &= -1 \end{aligned}$$

Critical Points

To find the critical points, we find where the first derivative equal to zero or does not exist.

$$\begin{aligned} r'(x) &= 0 \\ \text{numerator} &= 0 \\ 500 - q &= 0 \\ q &= 500 \end{aligned}$$

$$\begin{aligned} f'(x) &\text{ does not exist} \\ \text{denominator} &= 0 \\ 1 &= 0 \\ &\text{Always False} \\ &\text{No Solution} \end{aligned}$$

Maximum or Minimum

To find out whether $q = 500$ is a maximizer or minimizer, we will use the second derivative test. so we check

$$r''(500) = -1 < 0$$

So $q = 500$ is a local maximizer with maximizer price $p = 500 - 0.5(500) = 250$ and maximum revenue $r(500) = 125000$.

Exercise

(Old Final Exam Question) A company manufacturer sells q phones per week. The weekly demand is given by

$$p = -50q + 300$$

Find the selling price that maximizes the revenue.

Example

(Old Final Exam Question) For a certain product, the demand function is $p = 100e^{-0.01q}$, where q is the number of items. Find the selling price that maximizes the revenue.

Solution:
We find the revenue function to be

$$r = qp = q(100e^{-0.01q})$$

We find the derivatives first which are

$$r' = p + qp'$$
$$r' = 100e^{-0.01q} + 100qe^{-0.01q}(-0.01)$$
$$r' = 100e^{-0.01q} - qe^{-0.01q}$$
$$r' = (100 - q)e^{-0.01q} = \frac{100 - q}{e^{0.01q}}$$
$$r'' = -e^{-0.01q} + (100 - q)e^{-0.01q}(-0.01)$$

Critical Points

To find the critical points, we find where the first derivative equal to zero or does not exist.

$$r'(x) = 0$$
$$\text{numerator} = 0$$
$$(100 - q) = 0$$
$$q - 100 = 0$$
$$q = 100$$

$$f'(x) \text{ does not exist}$$
$$\text{denominator} = 0$$
$$e^{0.01q} = 0$$
$$0.01q* = \ln 0 = -\infty$$

No Solution

Maximum or Minimum

To find out whether $q = 100$ is a maximizer or minimizer, we will use the second derivative test. so we check

$$r''(100) = \;< 0$$

So $q = 100$ is a local maximizer with maximizer price $p = 100e^{-0.01(100)} = 100e^{-1}$ and maximum revenue $r(100) =$.

Example

(Old Exam Question) Assume the total cost of producing q units of a product is given by

$$c = q^3 - 24q^2 + 250q + 338$$

Find the level of production that minimizes the cost.

Solution:
We find the derivatives first which are

$$c' = 3q^2 - 48q + 250$$
$$c'' = 6q - 48$$

Critical Points

To find the critical points, we find where the first derivative equal to zero or does not exist.

$$c'(x) = 0$$
$$\text{numerator} = 0$$
$$3q^2 - 48q + 250 = 0$$

No Solution

$$c'(x) \text{ does not exist}$$
$$\text{denominator} = 0$$
$$1 = 0$$

Always False

No Solution

Maximum and Minimum

Since there is no local minimum, we take $q = 0$ as a minimizer with minimum cost $c = c(0) = 338$.

Example

(Old Exam Question) A manufacturer produces water bottles that sell for 400 BD each. The total cost of producing q bottles is given by the function $c(q) = 0.012q^2 - 29.6q + 5080$. What is the number of bottles that should be made to maximize the profit.

Solution:
We find the net profit function $N(q)$ to be

$$N = \text{Total Revenue} - \text{Total Cost} = 400q - (0.012q^2 - 29.6q + 5080)$$

We find the derivatives first which are

$$N' = 400 - 0.024q + 29.6$$
$$N'' = -0.024$$

Critical Points

To find the critical points, we find where the first derivative equal to zero or does not exist.

$$N'(x) = 0$$
$$\text{numerator} = 0$$
$$400 - 0.024q + 29.6 = 0$$
$$q = 17900$$

$$N'(x) \text{ does not exist}$$
$$\text{denominator} = 0$$
$$1 = 0$$

Always False

No Solution

Maximum or Minimum

To find out whether $q = 17900$ is a maximizer or minimizer, we will use the second derivative test. so we check

$$N''(17900) = \ < 0$$

So $q = 17900$ is a local maximizer with maximum net profit $N(17900) = .$

Example

(Old Exam Question) The demand function for a product is $p = 700 - 2q$ and the average cost per unit for producing q units is $\overline{c}(q) = q + 100 + \frac{1000}{q}$. Find the selling price that maximizes the profit and find the maximum profit.

Solution:

We find the net profit function $N(q)$ to be

$$\begin{aligned} N &= \text{Total Revenue} - \text{Total Cost} = qp + q\overline{c}(q) \\ &= 700q - 2q^2 - q^2 - 100q - 1000 \\ &= -3q^2 + 600q - 1000 \end{aligned}$$

We find the derivatives first which are

$$\begin{aligned} N' &= -6q + 600 \\ N'' &= -6 \end{aligned}$$

Critical Points

To find the critical points, we find where the first derivative equal to zero or does not exist.

$$\begin{aligned} N'(x) &= 0 \\ \text{numerator} &= 0 \\ -6q + 600 &= 0 \\ q &= 100 \end{aligned}$$

$N'(x)$ does not exist

$$\begin{aligned} \text{denominator} &= 0 \\ 1 &= 0 \end{aligned}$$

Always False

No Solution

Maximum or Minimum

To find out whether $q = 100$ is a maximizer or minimizer, we will use the second derivative test. so we check

$$N''(100) = -6 < 0$$

So $q = 100$ is a local maximizer with a maximizer price $p = 500$ and maximum net profit $N(100) = 29000$.