Section 14.3
Initial value problems

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MATHS 104: Mathematics for Business II

Differential equations

Definition

The *Initial value problem* is the problem of finding the function y given the derivative y' and *initial condition* $y(a) = b$.

Idea: Integrate y' and find the general antiderivative and then substitute $x = a$ and $y = b$ to find the function y.

Note: The initial value problem is subclass of a bigger problem in mathematics called solving differential equations.

Example

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = 6x^2 - 8x + 12 \text{ and } y(0) = 2$$

Solution: We integrate to find y.

$$
\begin{aligned}
y &= \int (6x^2 - 8x + 12)\, dx \\
y &= 2x^3 - 4x^2 + 12x + C \\
2 &= y(0) \\
2 &= 2(0)^3 - 4(0)^2 + 12(0) + C \\
2 &= C \\
y &= 2x^3 - 4x^2 + 12x + 2
\end{aligned}
$$

Exercise

(Old Exam Question) If y satisfies the given condition, find $y(x)$.

$$\frac{dy}{dx} = 9x^2 - 4x + 5 \text{ and } y(-1) = 0.$$

Example

If y satisfies the given condition, find $y(x)$.

$$y'' = 6x + 4 \text{ and } y'(0) = 1, y(0) = 5$$

Solution: We integrate to find y.

$$y' = \int (x+1)\, dx$$

$$y' = 3x^2 + 4x + C$$

$$1 = y'(0)$$

$$1 = 3(0)^2 + 4(0) + C \rightarrow C = 1$$

$$y' = 3x^2 + 4x + 1$$

$$y = \int (3x^2 + 4x + 1)\, dx$$

$$y = x^3 + 2x^2 + x + D$$

$$5 = y(0) \rightarrow D = 5$$

$$y = x^3 + 2x^2 + x + 5$$

Example

(Old Exam Question) The marginal revenue is $r'(q) = 240 - 4q$. What price will be paid for each unit when the level of production is $q = 5$ units?

Solution: We integrate to find the revenue r first.

$$r = \int (240 - 4q)\, dq$$

$$r = 240q - 2q^2 + C$$

$$0 = r(0) \qquad \textbf{because no revenue if no quanitity is sold}$$

$$0 = 240(0) - 2(0)^2 + C$$

$$0 = C$$

$$r = 240q - 2q^2$$

Now we find the price which is

$$p = \frac{r}{q} = \frac{240q - 2q^2}{q} = 240 - 2q$$

$$p = 240 - 5 = 230$$

Example

(Old Exam Question) The marginal revenue is $r'(q) = 800 + 2q - 0.4q^3$. Find the revenue function.

Solution: We integrate to find the revenue r first.

$$r = \int (800 + 2q - 0.4q^3)\, dq$$

$$r = 800q + q^2 - 0.1q^4 + C$$

$$0 = r(0) \qquad \textbf{because no revenue if no quanitity is sold}$$

$$0 = C$$

$$r = 800q + q^2 - 0.1q^4$$

Example

(Old Exam Question) The marginal cost is $c'(q) = 0.4q^3 - 0.01q + 10$ and the fixed cost is 250 BD. find the total cost of producing 10 units.

Solution: We integrate to find the cost c first.

$$c(q) = \int (0.4q^3 - 0.01q + 10)\, dq$$

$$c(q) = 0.1q^4 + 0.005q^2 + 10q + C$$

$250 = c(0)$ **You have to pay the fixed cost even if no quanitity is so**

$$250 = C$$

$$c = 0.1q^4 + 0.005q^2 + 10q + 250$$

$$c(10) = 0.1(10)^4 + 0.005(10)^2 + 10(10) + 250$$

$$c(10) =$$

Example

(Old Final Exam Question) The marginal cost is $c'(q) = 10 + q + \frac{100}{q^2}$ and the cost to produce 100 units is 10000 BD. find the cost function $c(q)$.

Solution: We integrate to find the cost c first.

$$
\begin{aligned}
c(q) &= \int (10 + q + \frac{100}{q^2})\, dq \\
c(q) &= 10q + \frac{1}{2}q^2 - \frac{100}{q} + C \\
10000 &= c(100) \\
10000 &= 10(100) + \frac{1}{2}(100)^2 - \frac{100}{100} + C \\
4001 &= C \\
c(q) &= 10q + \frac{1}{2}q^2 - \frac{100}{q} + 4001
\end{aligned}
$$