Section 14.5
Techniques of Integration

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MATHS 104: Mathematics for Business II

Motivation

Example

Find $\int 3^x \, dx$

Solution: Since this is not a basic integral, we are looking for a good substitution. We are looking for an inner function with almost the derivative is somewhere in the integral. Let

$$u = 3^x$$

$$du = 3^x \ln 3 dx \rightarrow dx = \frac{du}{3^x \ln 3}$$

$$\begin{aligned} \int 3^x \, dx &= \int u \, \frac{du}{3^x \ln 3} \\ &= \frac{1}{\ln 3} \int u \, \frac{du}{u} = \frac{1}{\ln 3} \int 1 \, du \\ &= \frac{1}{\ln 3} u + C = \frac{1}{\ln 3} 3^x + C \end{aligned}$$

Integral of the exponential function

Theorem

$\int a^x \, dx = \frac{1}{a} a^x + C$

Proof: Since this is not a basic integral, we are looking for a good substitution. We are looking for an inner function with almost the derivative is somewhere in the integral. Let

$$u = a^x$$

$$du = a^x \ln a dx \to dx = \frac{du}{a^x \ln a}$$

$$\begin{aligned}
\int a^x \, dx &= \int u \, \frac{du}{a^x \ln a} \\
&= \frac{1}{\ln a} \int u \, \frac{du}{u} = \frac{1}{\ln a} \int 1 \, dx \\
&= \frac{1}{\ln a} u + C = \frac{1}{\ln a} a^x + C
\end{aligned}$$

Elementary integration formula

1. $\int k\, dx = kx + C.$

2. $\int x^n\, dx = \frac{1}{n+1} x^{n+1} + C \qquad n \neq -1.$

3. $\int x^{-1}\, dx = \int \frac{1}{x}\, dx = \ln x + C \qquad x > 0.$

4. $\int e^x\, dx = e^x + C.$

   $\int e^{kx}\, dx = \frac{1}{k} e^{kx} + C.$

5. $\int a^x\, dx = \frac{1}{\ln a} a^x + C.$

6. $\int kf(x)\, dx = k \int f(x)\, dx\ .$

7. $\int (f(x) + g(x))\, dx = \int f(x)\, dx + \int g(x)\, dx\ .$

Example

Find $\int x2^{4-x^2}\,dx$

Solution: Since this is not a basic integral, we are looking for a good substitution. We are looking for an inner function with almost the derivative is somewhere in the integral. Let

$$u = 4 - x^2$$

$$du = -2x dx \rightarrow dx = \frac{du}{-2x}$$

$$\begin{aligned}\int x2^{4-x^2}\,dx &= \int x2^{u}\,\frac{du}{-2x}\\ &= \frac{1}{-2}\int 2^{u}\,du\\ &= \frac{1}{-2\ln 2}2^{u} + C\\ &= \frac{1}{-2\ln 2}2^{4-x^2} + C\end{aligned}$$

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