Section 14.9 Area between curves 2 Lectures

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MATHS 104: Mathematics for Business II

Idea: To find the area between curves we have to do the following:

- Sketch the graph of each curve.
- Ind the intersection points.
- Integrate

 $\int^{b} (top - bottom) dx$

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = 17 - x^2$, $y = x^2 - 1$.

$$17 - x^{2} = x^{2} - 1$$

$$18 = 2x^{2}$$

$$2x^{2} - 18 = 0$$

$$x = -3 \text{ or } x = 3$$

Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area = $\int_{-3}^{3} ((17 - x^{2}) - (x^{2} - 1)) dx$
= $\int_{-3}^{3} (18 - 2x^{2}) dx$
= $\left[18x - \frac{2}{3}x^{3}\right]_{-3}^{3}$
= 72

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = x^2 + 1$, y = 2x + 1.

$$x^{2} + 1 = 2x + 1$$
$$x^{2} = 2x$$
$$x^{2} - 2x = 0$$
$$x = 0 \text{ or } x = 2$$

Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area =
$$\int_{0}^{2} ((2x + 1) - (x^{2} + 1)) dx$$
$$= \int_{0}^{2} (2x - x^{2}) dx$$
$$= \left[x^{2} - \frac{1}{3}x^{3}\right]_{0}^{2}$$
$$= \frac{4}{3}$$

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = e^x$ from x = 1 to x = 3.

Solution:

Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area = $\int_{1}^{3} (e^{x}) dx$
= $\int_{1}^{3} (e^{x}) dx$
= $[e^{x}]_{1}^{3}$
= $e^{3} - e^{1}$

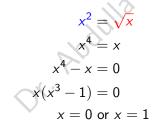
(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = 4 - x^2$, $y = 2x^2 + 1$.

$$4 - x^{2} = 2x^{2} + 1$$
$$3 = 3x^{2}$$
$$3x^{2} - 1 = 0$$
$$x = -1 \text{ or } x = 1$$

Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area = $\int_{-1}^{1} ((4 - x^{2}) - (2x^{2} + 1)) dx$
= $\int_{-1}^{1} (3 - 3x^{2}) dx$
= $[3x - x^{3}]_{-1}^{1}$
= 4

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = x^2$, $y = \sqrt{x}$.



Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area =
$$\int_{0}^{1} (\sqrt{x}) - (x^{2}) dx$$
$$= \int_{0}^{1} (\sqrt{x} - x^{2}) dx$$
$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^{3}\right]_{0}^{1}$$
$$= \frac{1}{3}$$

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = x^2 + x - 2$ and x-axis.

Solution: Recall that the *x*-axis is represented by the equation y = 0. First we find the intersection points, so have

$$x^{2} + x - 2 = 0$$

 $x^{2} + x - 2 = 0$
 $x = -2 \text{ or } x = 1$

Area =
$$\int_{a}^{b} (top - bottom) dx$$

Area = $\int_{-2}^{1} ((0) - (x^{2} + x - 2)) dx$
= $\int_{-2}^{1} (-x^{2} - x + 2) dx$
= $\left[-\frac{1}{3}x^{3} - \frac{1}{2}x^{2} + 2x \right]_{-2}^{1}$
= $\frac{9}{2}$

(Old Final Exam Question) Sketch and find the area of the region bounded by the curves $y = 17 - x^2$, $y = x^2 - 1$ from x = -4 to x = 1.

$$17 - x^{2} = x^{2} - 1$$

$$18 = 2x^{2}$$

$$2x^{2} - 18 = 0$$

$$x = -3 \text{ or } x = 3$$

Area =
$$\int_{a}^{b} (\text{top} - \text{bottom}) dx$$

Area = $\int_{-4}^{-3} ((x^{2} - 1) - (17 - x^{2})) dx + \int_{-3}^{1} ((17 - x^{2}) - (x^{2} - 1)) dx$
= $\int_{-4}^{-3} (2x^{2} - 18) dx + \int_{-3}^{1} (18 - 2x^{2}) dx$
= $\left[\frac{2}{3}x^{3} - 18x\right]_{-4}^{-3} + \left[18x - \frac{2}{3}x^{3}\right]_{-3}^{1}$
= 60