Section 15.4
Average Value of a function
0.5 Lecture

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MATHS 104: Mathematics for Business II

The Average Value of a function

Definition

The *average value of a function* $f(x)$ over the interval $[a, b]$ is denoted by $\overline{f}$ and is given by

$$\overline{f} = \frac{1}{b-a} \int_a^b f(x)\, dx$$

Example

The total cost of printing q dictionaries is $c(q) = 20000q + 5q^3$. Find the average value of the total cost over the interval $[0, 10]$.

Solution: The average cost is given by

$$\overline{c} = \frac{1}{10-0}\int_0^{10} c(q)\,dq = \frac{1}{10-0}\int_0^{10} (20000q + 5q^3)\,dq$$

$$\left[10000q^2 + \frac{5}{4}q^4\right]_0^{10} = \frac{1}{10}\left[10000q^2 + \frac{5}{4}q^4\right]_0^{10}$$

$$\frac{1}{10}\left(10000(10)^2 + \frac{5}{4}(10)^4\right) - \frac{1}{10}\left(10000(0)^2 + \frac{5}{4}(0)^4\right) = 101250$$

Example

(Old Final Exam Question) For the cost function $c(q) = 200 + 20q + 0.222q^2$, find the average cost on the interval from $q = 2$ to $q = 22$.

Solution: The average cost is given by

$$\begin{aligned}
\frac{1}{22-2}\int_2^{22} c(q)\,dq &= \frac{1}{20}\int_2^{22}(200 + 20q + 0.222q^2)\,dx \\
&= \frac{1}{10}\left[200q + 10q^2 + \frac{0.222}{3}q^3\right]_2^{22} \\
&= \frac{1}{20}\left(200(22) + 10(22)^2\frac{0.222}{3}(22)^3\right) \\
&- \frac{1}{20}\left(200(2) + 10(2)^2\frac{0.222}{3}(2)^3\right) = 497.37
\end{aligned}$$

Exercise

(Old Final Exam Question) For the cost function $c(q) = 160 + 8q + 0.12q^2$, find the average cost on the interval from $q = 10$ to $q = 20$.

Example

For the cost function $c(q) = 6q\sqrt[3]{100+q^2}$. Find the average cost on the interval from $q = 5$ to $q = 30$.

Solution:

The average cost is given by

$$\frac{1}{30-5}\int_5^{30} c(q)\,dq = \frac{1}{25}\int_5^{30} 6q\sqrt[3]{100+q^2}\,dq$$

Since this is not a basic integral, we are looking for a good substitution. We are looking for an inner function with almost the derivative is somewhere in the integral. Let

$$u = 100 + q^2$$

$$du = 2q\,dx \rightarrow dq = \frac{du}{2q}$$

$$\text{if } q = 5, \text{then } u = 125$$

$$\text{if } q = 30, \text{then } u = 1000$$

$$\frac{1}{25}\int_5^{30} 6q\sqrt[3]{100+q^2}\,dq = \frac{1}{25}\int_{125}^{1000} 6q\sqrt[3]{u}\,\frac{du}{2q} = \frac{3}{25}\int_{125}^{1000} (u)^{\frac{1}{3}}\,du$$

$$= \left[\frac{9}{100}(u)^{\frac{4}{3}}\right]_{125}^{1000}$$

$$= \frac{9}{100}\left((1000)^{\frac{4}{3}}\right) - \frac{9}{100}\left((125)^{\frac{4}{3}}\right)$$

$$= \frac{3375}{4}$$