Section 17.2
Applications of Partial Derivative
0.25 Lecture

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MATHS 104: Mathematics for Business II

The Marginal Cost

Definition

Given the cost $c = c(x, y)$ as a joint function of producing x quantities of item A and y quantities of item B.

1. The **partial marginal cost with respect** to x is given by $\frac{\partial c}{\partial x}$.
2. The **partial marginal cost with respect** to y is given by $\frac{\partial c}{\partial y}$.

Example

If $c = 7x + 0.3y^2 + 2y + 400$. Find the marginal cost with respect to y for $x = 20$ and $y = 30$.

Solution:

$$\frac{\partial c}{\partial y} = 0.6y + 2$$

$$\frac{\partial c}{\partial y}(20, 30) = 20$$

Competitive and Complementary Products

Sometimes two products may be related such that change in price of one of them affect the demand for the other. For example, Milk and yoghurt.

$q_1 = f(p_1, p_2)$ Demand function for the first product

$q_2 = g(p_1, p_2)$ Demand function for the second product

Definition

1. If
$$\frac{\partial q_1}{\partial p_2} > 0 \text{ and } \frac{\partial q_2}{\partial p_1} > 0$$
Then we say that the two products are **competitive**, i.e., an increase of the price in the second item, increase the demand for the first item and vice versa. Example, milk and yoghurt.
2. If
$$\frac{\partial q_1}{\partial p_2} < 0 \text{ and } \frac{\partial q_2}{\partial p_1} < 0$$
Then we say that the two products are **complementary**, i.e., an increase of the price in the second item, decrease the demand for the first item and vice versa. Example cars and gasoline.

Example

If $q_1 = 1500 - 40p_1 + 3p_2$ and $q_2 = 900 + 5p_1 - 20p_2$. Determine whether these two products are complementary or competitive or neither.

Solution:

$$\frac{\partial q_1}{\partial p_2} = 3 > 0$$
$$\frac{\partial q_2}{\partial p_1} = 5 > 0$$

Hence the two products are competetive.