Section 17.4
Higher–order Partial Derivative
0.25 Lecture

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The Multivariable function

Definition

Let $f(x, y)$ be a two variables function, then the **second–order partial derivatives** are

$$f_{xx} \text{ means } (f_x)_x \qquad \frac{\partial^2 f}{\partial x^2}$$

$$f_{xy} \text{ means } (f_x)_y \qquad \frac{\partial^2 f}{\partial x \partial y}$$

$$f_{yx} \text{ means } (f_y)_x \qquad \frac{\partial^2 f}{\partial y \partial x}$$

$$f_{yy} \text{ means } (f_y)_y \qquad \frac{\partial^2 f}{\partial y^2}$$

Example

If $f(x, y) = x^2y + 3xy$. Find all the second–order partial derivative.

Solution:

$$
\begin{aligned}
f_x &= 2xy + 3y \\
f_{xx} &= 2y \\
f_{xy} &= 2x + 3 \\
f_y &= x^2 + 3x \\
f_{yy} &= 0 \\
f_{yx} &= 2x + 3
\end{aligned}
$$

Example

If $f(x,y) = x^3y^2 + x^2y - x^2y^2$. Find $f_{xxy}(2,3)$ and $f_{xyx}(2,3)$

Solution:

$$
\begin{aligned}
f_x &= 3x^2y^2 + 2xy - 2xy^2 \\
f_{xx} &= 6xy^2 + 2y - 2y^2 \\
f_{xyy} &= 12xy + 2 - 4y \rightarrow f_{xxy}(2,3) = 62 \\
f_{xy} &= 6x^2y + 2x - 4xy \\
f_{xyx} &= 12xy + 2 - 4y \rightarrow f_{xxy}(2,3) = 62
\end{aligned}
$$