Section 17.6
Local Extrema
0.25 Lecture

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Local Extrema

Definition

Let $f(x, y)$, the critical points of f are the points where $f_x = 0$ and $f_y = 0$. To check the local maximum and local minimum we apply the second derivative test.

1. If $f_{xx}f_{yy} - (f_{xy})^2 > 0$ and $f_{xx} > 0$, then we have a local minimum.
2. If $f_{xx}f_{yy} - (f_{xy})^2 > 0$ and $f_{xx} < 0$, then we have a local maximum.
3. If $f_{xx}f_{yy} - (f_{xy})^2 < 0$, then we have a **saddle** point.
4. If $f_{xx}f_{yy} - (f_{xy})^2 = 0$, then the second derivative test is **inconclusive**.

Example

If $f(x,y) = x^3 + 3xy^2 + 3y^2 + 2$. Find the critical point.

Solution:

$$f_x = 3x^2 + 3y^2$$
$$f_y = 6xy + 6y$$

So $x = 0$ and $y = 0$ and hence we have $(0,0)$ is a critical point.

Example

If $f(x,y) = x^2 + \frac{1}{3}y^3 + 2xy - 8y + 6$. Find the relative extrema.

Solution:

$$f_x = 2x + 2y \to x = -y$$
$$f_y = y^2 + 2x - 8 \to y^2 - 2y - 8 = 0$$

So $y = -2$ and $y = 4$ and hence we have $(2, -2)$ and $(-4, 4)$ are critical points. To check which is maximum or minimum, we use the second derivative test, hence we check

$$f_{xx} = 2$$
$$f_{yy} = 2y$$
$$f_{xy} = 2$$
$$f_{xx}f_{yy} - (f_{xy})^2 = 2y - 4$$

Therefore, f has at $(-4, 4)$ a local minimum and a saddle point at $(2, -2)$.