

Section 4.4

Basis

Dr. Abdulla Eid

College of Science

MATHS 211: Linear Algebra

Goal:

- 1 Define a basis of a vector space.

Dr. Abdulla Eid

Definition 1

Let V be a vector space. $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is called a **basis** if

- 1 \mathcal{B} is linearly independent.
- 2 \mathcal{B} spans V .

Example 2

Determine whether the vectors $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ form a basis for \mathbb{R}^3 or not.

This is called the *standard* basis for \mathbb{R}^3 .

Example 3

Show that the vectors $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$ form a basis for \mathbb{R}^3 .

Dr. Abdulla L

Example 4

Determine whether the vectors $\mathbf{v}_1 = 2 - 4X + X^2$, $\mathbf{v}_2 = 3 + 2X - X^2$, $\mathbf{v}_3 = 1 + 6X - 2X^2$ form a basis for P_2 .

Dr. Abdulla Eid

Example 5

Determine whether the vectors $\mathbf{v}_1 = \begin{pmatrix} 3 & 4 \\ 3 & -4 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,
 $\mathbf{v}_3 = \begin{pmatrix} 0 & -8 \\ -12 & -2 \end{pmatrix}$ form a basis for $\text{Mat}(2, 2, \mathbb{R})$.

Dr. Abdulla

Theorem 6

Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ be a basis for V , then every vector v in V can be written uniquely in the form

$$v = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$$

Definition 7

Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ be basis for V and let w in V with

$$\mathbf{v} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_n\mathbf{v}_n$$

The scalars c_1, \dots, c_n are called **coordinates** of \mathbf{v} in terms of \mathcal{B} . The vector $(c_1, \dots, c_n) \in \mathbb{R}^n$ is called the **coordinate vector of \mathbf{v} relative to \mathcal{B}** . It is denoted by

Example 8

Let $\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \right\}$ be a basis for some subspace of \mathbb{R}^3 .

Find the coordinate vector of

① $\begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix}$.

② $\begin{pmatrix} 5 \\ -12 \\ 3 \end{pmatrix}$.

Example 9

Let $\mathcal{B} = \{1 + X, 1 + X^2, X + X^2\}$ be a basis for some subspace of \mathbb{P}_2 .
Find the coordinate vector of

① $3 - X - 2X^2$

Dr. Abdulla E.

Example 10

Let $\mathcal{B} = \left\{ \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$ be a basis for some subspace of $\text{Mat}(2, 2, \mathbb{R})$. Find the coordinate vector of

① $\begin{pmatrix} -5 & 4 \\ 1 & - + + + + 1 \end{pmatrix}$

Dr. Abdulla