

Section 1.2

Row Operations

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MATHS 211: Linear Algebra

Elementary Row Operations

What are the *elementary row operations* on a matrix?

- 1 Interchanging any two rows ($R_i \leftrightarrow R_j$).
- 2 Multiplying (dividing) a row by a non-zero number ($R_i \rightarrow cR_i$).
- 3 Add a multiple of a row to another row ($R_i \rightarrow R_i + cR_j$).

Example

Consider the following matrix

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 5 & 1 \\ 3 & 0 & -2 \end{pmatrix}$$

Perform $R_3 \rightarrow R_3 + 2R_1$.

Solution:

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 5 & 1 \\ 3 + 2(1) & 0 + 2(0) & -2 + 2(2) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 5 & 1 \\ 5 & 0 & 2 \end{pmatrix}$$

Goal of the elementary row operations:

We want to reach a matrix in **reduced row echelon form (RREF)**, which is a matrix that satisfy the following properties:

- 1 All zero-rows are at the bottom of the matrix.
- 2 Each non-zero row has a leading 1's (called **pivot**).
- 3 The pivots start from left to right (up to down).
- 4 all entries in the pivot columns are zeros.

Example

Which of the following matrices are reduced matrix?

$$\begin{pmatrix} 1 & 3 & 0 & 5 & 1 \\ 0 & 0 & 1 & 2 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Example

Reduce the matrix

$$\begin{pmatrix} 0 & -3 & 0 & 2 \\ 1 & 5 & 0 & 2 \end{pmatrix}$$

Solution:

$$\begin{aligned} & \begin{pmatrix} 0 & -3 & 0 & 2 \\ 1 & 5 & 0 & 2 \end{pmatrix}, & R_1 \leftrightarrow R_2 \\ & \begin{pmatrix} 1 & 5 & 0 & 2 \\ 0 & -3 & 0 & 2 \end{pmatrix}, & R_2 \rightarrow \frac{1}{-3}R_2 \\ & \begin{pmatrix} 1 & 5 & 0 & 2 \\ 0 & 1 & 0 & \frac{2}{-3} \end{pmatrix}, & R_1 \rightarrow R_1 - 5R_2 \\ & \begin{pmatrix} 1 - 5(0) & 5 - 5(1) & 0 - 5(0) & 2 - 5(\frac{2}{-3}) \\ 0 & 1 & 0 & \frac{2}{-3} \end{pmatrix} \\ & \begin{pmatrix} 1 & 0 & 0 & \frac{16}{3} \\ 0 & 1 & 0 & \frac{2}{-3} \end{pmatrix} \end{aligned}$$

Example

Reduce the matrix

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & -2 & -8 \\ 5 & 0 & -5 \end{pmatrix}$$

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Example

Reduce the matrix

$$\begin{pmatrix} 3 & 3 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & -1 \\ 3 & 5 & 1 & 2 & 3 \end{pmatrix}$$

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