Section 4.0
Introduction to Chapter 4
The space $\mathbb{R}^2$

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MATHS 211: Linear Algebra

Goal:

1. Frame System.
2. Number of vectors.

Moon and the Earth!

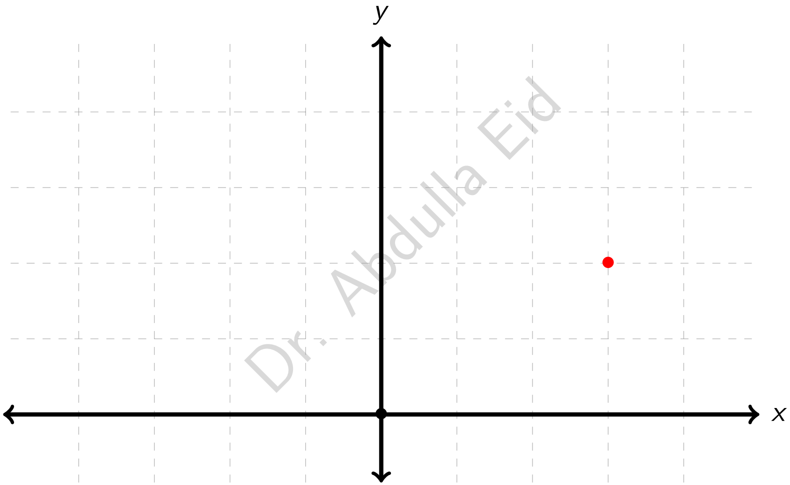
Describing a point!

Question: How could you describe the red point?

Answer: You Can NOT ! Lack of coordinates.

Standard frame reference

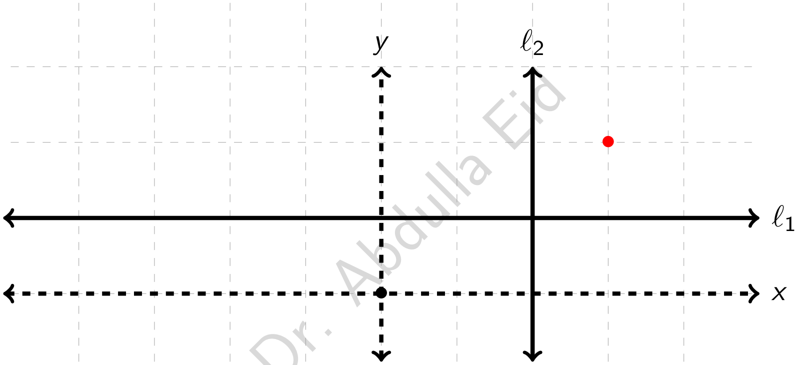
Question: Can you now describe this point?



Answer: Yes, start with the origin with three steps to the right and two steps up , i.e., $x_0 = 3, y_0 = 2$

Any frame reference

Question: Can you now describe this point?

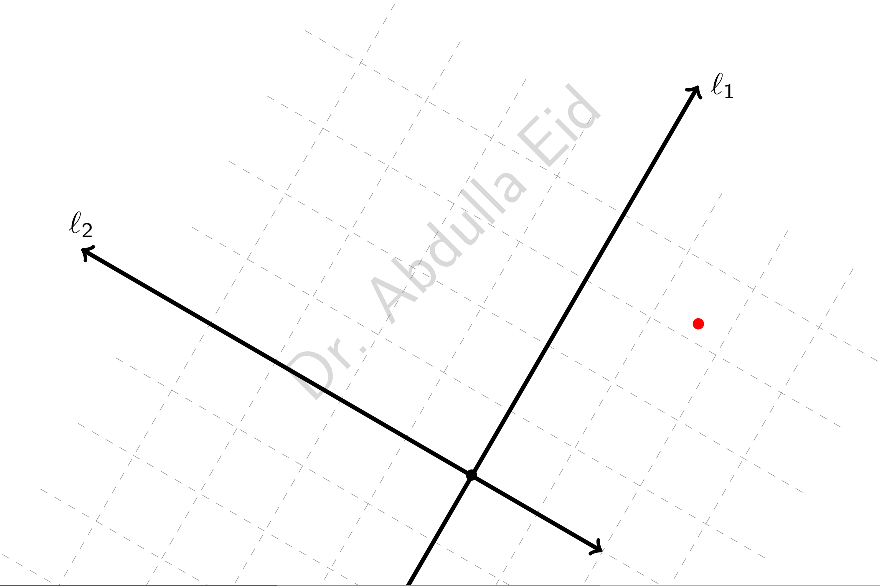


Answer: Yes, start with the new origin with one step to the right and one step up , i.e., $x_{\text{new}} = 1, y_{\text{new}} = 1$.

Question: Is there any relation to get $x_{\text{new}} =???, y_{\text{new}} =???$ by knowing x_0, y_0 for any point?

Any frame reference that fixes the origin

Question: Can you now describe this point?



Answer: Yes, start at the origin parallel to the first line until you reach the point (3.2 steps) and then in the direction of the the other line until you reach the point (1.6 steps).

Question: How can we describe the paragraph above as mathematical sentence?

Answer: $P = (3.2, 1.6)$ BUT maybe also $P = (1.6, 3.2)$. That lead to confusion.

Answer: $P = 3.2\ell_1 + 1.6\ell_2$ **Linear Combination**

So in any frame system, we need to be able to add lines, scalar multiply a line, and we must have an origin.
Question: Is there any relation to get $x_{\text{new}} =???$, $y_{\text{new}} =???$ by knowing x_0, y_0 for any point?

Position is relative

Facts:

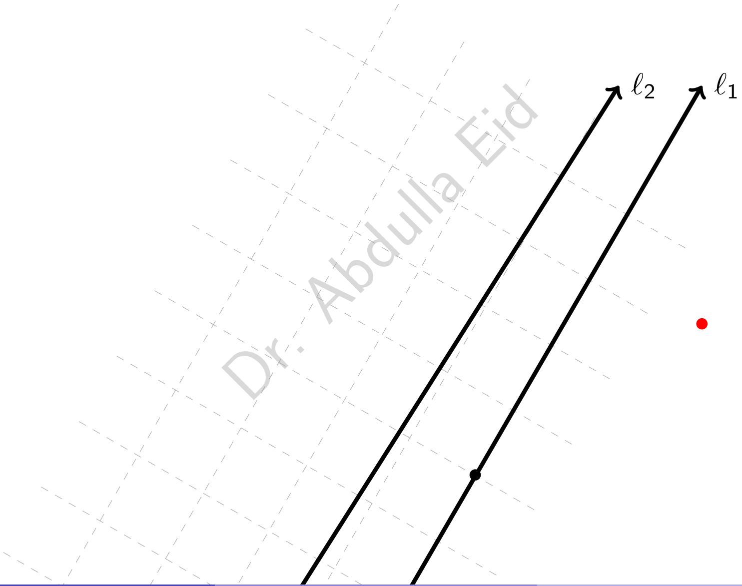
- To find the position of a point, you will need a frame system to tell you how to find that point.
- In each frame system we need to be able to add, scalar multiply, and we have an origin.
- There are many (in fact infinitely many) frame references that can be used.
- Each point has **coordinates** for each frame reference.
- There is one **standard** frame reference.
- Frame references are all related with each other and that is one of the task of linear algebra is to describe how they are related.

Questions

- Will any two lines give a frame reference?
- Can we have a frame reference with only one line? three or more lines?
- How can we name the lines in the frame reference in an efficient way?

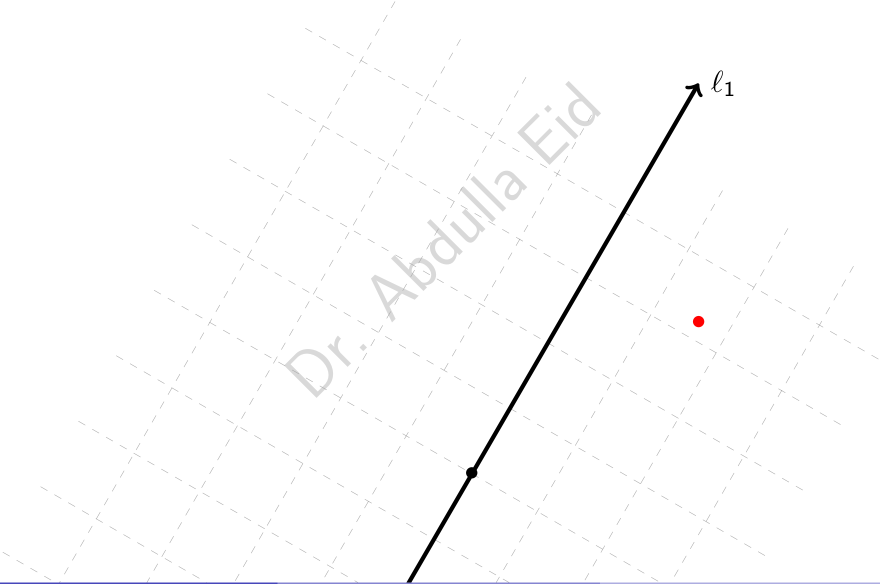
Parallel lines

Question: Can you now describe this point?



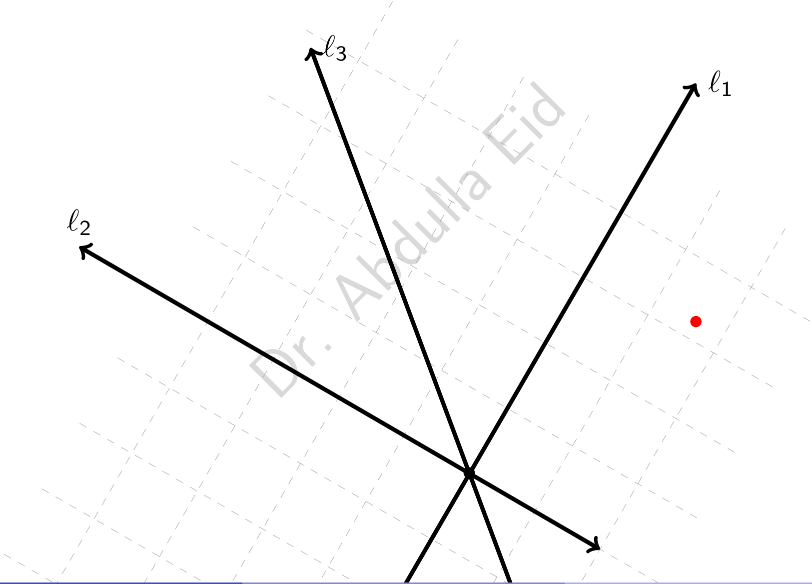
One line

Question: Can you now describe this point?



Three lines

Question: Can you now describe this point?



Naming of the lines, VECTORS!

- To define a line, we need to figure out two points on the line. One is the origin point $\mathbf{0}$ and the other point is $\mathbf{v}$.
- The most efficient way to name the lines in a frame reference is to think about them as **extension of a vector**. There will be many vectors (infinitely many) to describe the same line though.
- So the frame reference will be given mathematically as a set of two specific vectors, like $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ or $S = \{\mathbf{v}'_1, \mathbf{v}'_2\}$ and so on.
- Each point in the frame system is given by linear combinations of the lines in the frame system, i.e., $P = a\mathbf{v}_1 + b\mathbf{v}_2$ or $P = a'\mathbf{v}'_1 + b'\mathbf{v}'_2$ and so
- The set of all points that can be described by the vectors in the frame reference will be called the **spanning set**.

We will need to formulate all these in equations!

Matrix Multiplication

Given any frame reference $S = \{\mathbf{v}_1, \mathbf{v}_2\}$, how to relate $x_{\text{new}}, y_{\text{new}}$ with x_0, y_0?

$$\begin{pmatrix} x_{\text{new}} \\ y_{\text{new}} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$